Physics 221, April 6

Key Concepts:

- Harmonic motion
- The pendulum
- Damped and driven oscillation
- Traveling waves
- Standing waves

Harmonic motion

If the only force acting on an object with mass m is a Hooke's law force,

$$F = -kx$$

then the motion of the object is simple harmonic motion.

With x being the displacement from equilibrium we have

$x(t) = A\cos(\omega t + \phi)$,

$v(t) = -\omega A\sin(\omega t + \phi)$,

$a(t) = -\omega^2 A\cos(\omega t + \phi) = -\omega^2 x$.

$\omega = (k/m)^{1/2} = 2\pi f = 2\pi/T$.

A = amplitude

ω = angular frequency

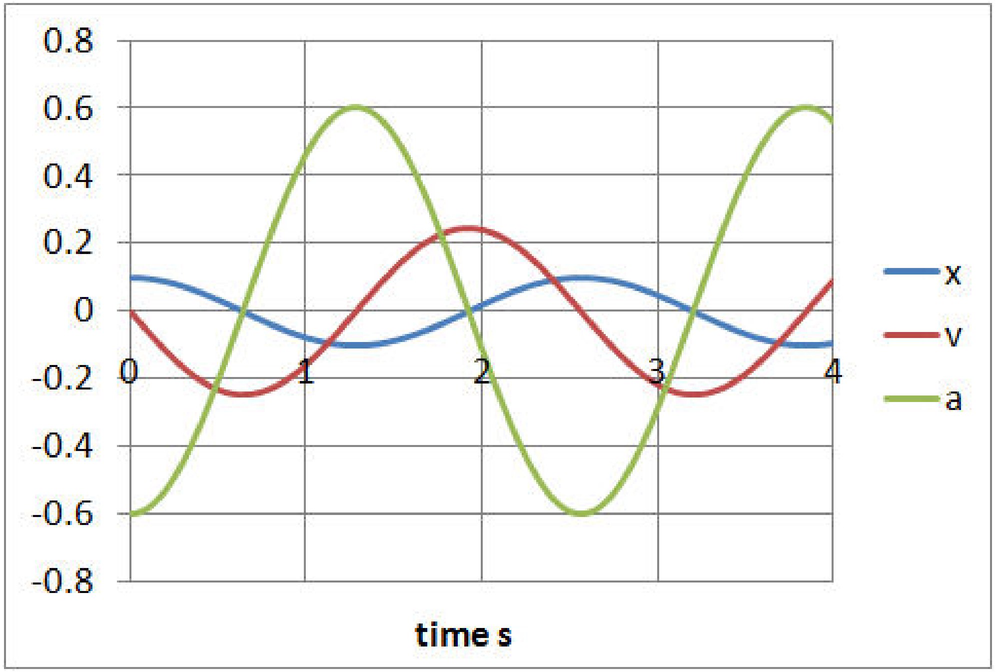
f = frequency

T = period

ϕ = phase constant

In terms of k and m:

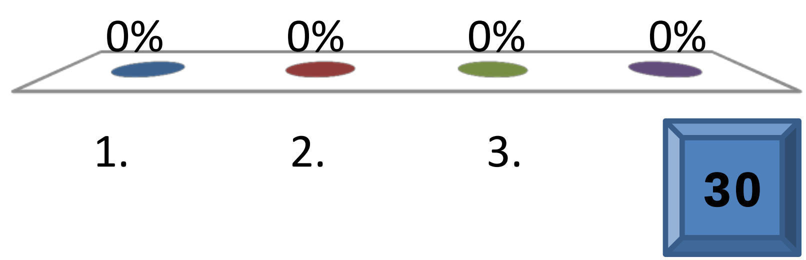
$$\omega = \sqrt{\frac{k}{m}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

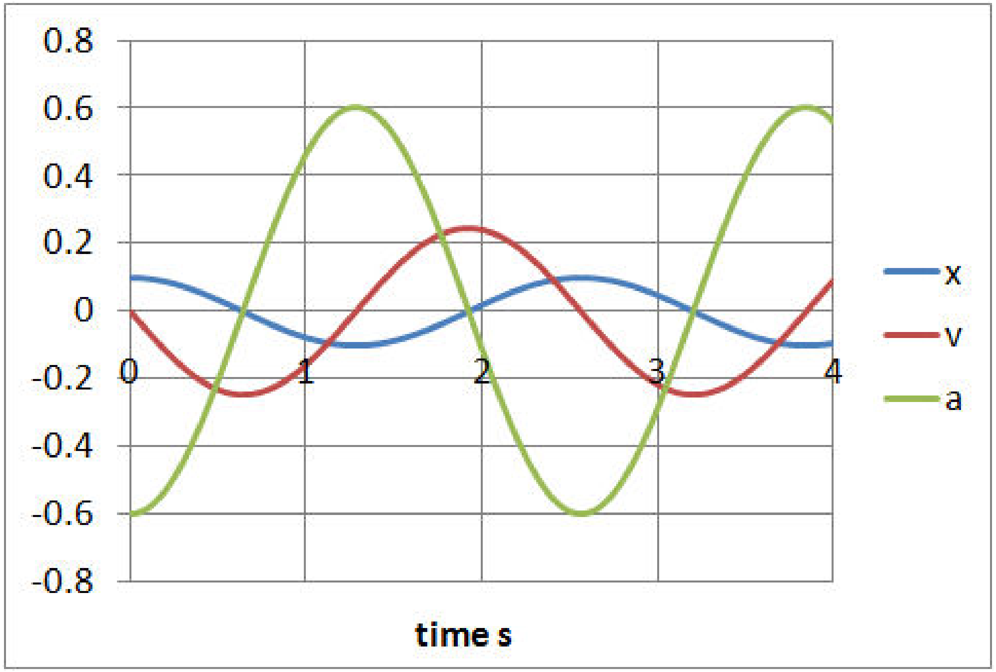


Position, velocity and acceleration of an oscillating mass on a spring are shown as a function of time.

What is the period of the oscillations?

1. ~1.5 s
2. ~2.5 s
3. ~4 s
4. ~0.4 s





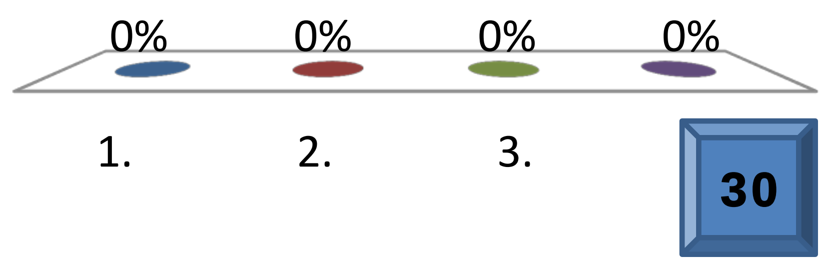
Position, velocity and acceleration of an oscillating mass on a spring are shown as a function of time.

If the mass is 0.2 kg, what is the spring constant in N/m?

Hint:

$T^2/(4\pi^2) = m/k$

1. ~6
2. ~3
3. ~1.2
4. ~0.5



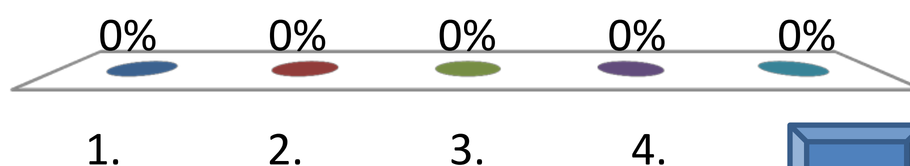
Demo:

Measure the frequency or period of a 0.1 kg mass oscillating on a spring.

What is the spring constant of this spring?

Hint:
$T^2/(4\pi^2) = m/k$

1. ~0.3 N/m
2. ~0.9 N/m
3. ~3 N/m
4. ~9 N/m
5. ~80 N/m



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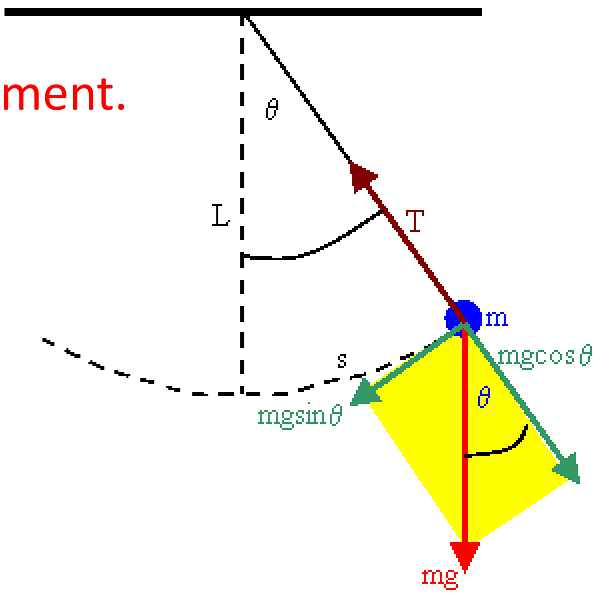
The simple pendulum

Hooke's law: $F = -kx$ → $x(t) = x_{max}\cos(\omega t + \phi)$, $\omega^2 = k/m$.
If the restoring force is proportional to the displacement from equilibrium, then the system executes simple harmonic motion.

What about a pendulum?
Displacement from equilibrium: $s = L\theta$.
Net force: $F = -mg\sin\theta$.
The restoring force is not proportional to the displacement.



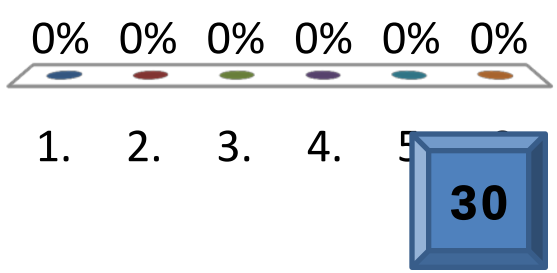
But for small displacements $\sin\theta \sim \theta$.
Then $F = -mg\theta$ or $F = -ks$, with $k = mg/L$.
Hooke's law → simple harmonic motion
$s(t) = s_{max}\cos(\omega t + \phi)$, $\omega^2 = k/m = g/L$

$T = 2\pi(L/g)^{1/2}$, $f = 1/T = (1/2\pi)(g/L)^{1/2}$.

For a simple pendulum the period of small oscillations is independent of the mass.

A simple pendulum of length L oscillates back and forth 10 times per second. By what factor do you have to change its length to make it oscillate back and forth only 5 times per second?

1. Increase the length by a factor of √2.
2. Increase the length by a factor of 2.
3. Increase the length by a factor of 4.
4. Decrease the length by a factor of √2.
5. Decrease the length by a factor of 2.
6. Decrease the length by a factor of 4.



A simple pendulum of length L oscillates back and forth 10 times per second. By what factor do you have to change its length to make it oscillate back and forth only 5 times per second?

- Original frequency: 10/s
- Original period: (1/10)s
- Desired frequency: 5/s
- Desired period: (1/5)s

You want to double the period. $T = 2\pi(L/g)^{1/2}$, $T^2 \propto L$.

Link: Pendulum Waves

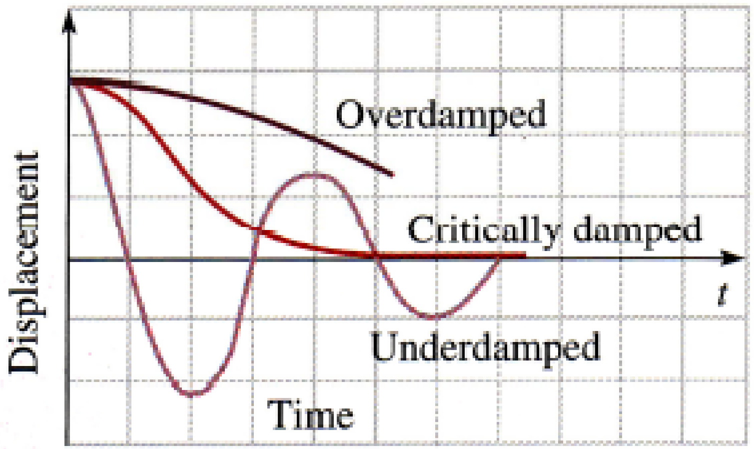
Damped oscillations

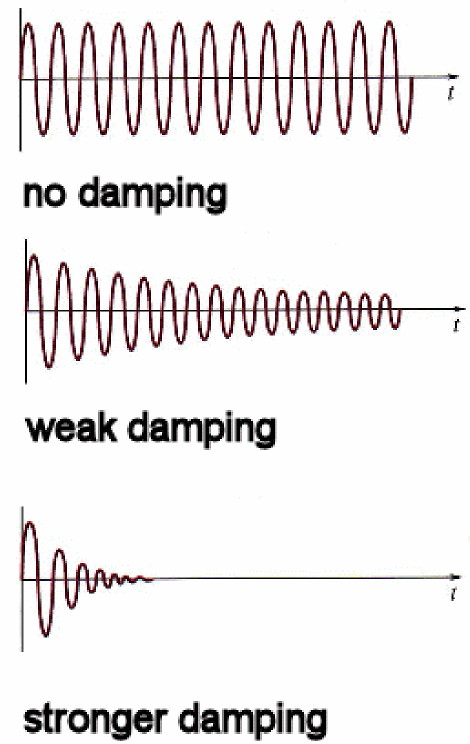
Free oscillations of macroscopic systems damp out.

If $F = -kx - bv$, then

$x(t) = A_0\exp(-bt/2m)\cos(\omega_{damp}t + \phi)$,

with $\omega_{damp}^2 = k/m - (b/2m)^2$.





The energy stored in an oscillating system is proportional to the square of the amplitude, it decreases as $E = E_0\exp(-bt/m)$.

$A \rightarrow A_0/2$ implies $E \rightarrow E_0/4$

Forced oscillations

Consider a damped harmonic oscillator acted on by a driving force $F_0\cos\omega_{ext}t$.

Net force acting on the oscillator: $F = F_0\cos\omega_{ext}t - kx - bv$.

Resulting motion: $x(t) = A\cos(\omega t + \phi)$, with $\omega = \omega_{ext}$.

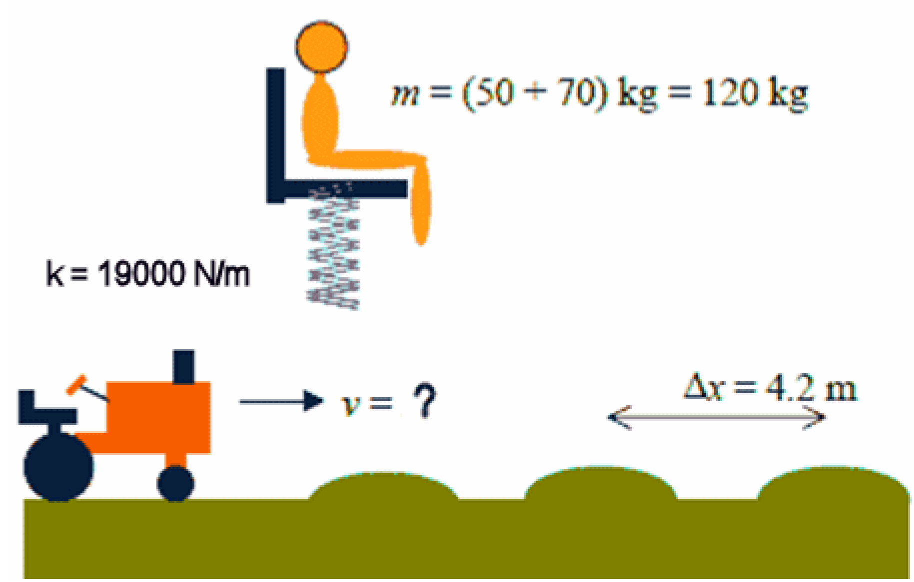
$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

The amplitude of forced oscillations is small unless the driving frequency is close to the natural frequency $\omega_0 = (k/m)^{1/2}$.

When the driving frequency is equal to the natural frequency the amplitude of the oscillations can be become very large. This is called **resonance.**

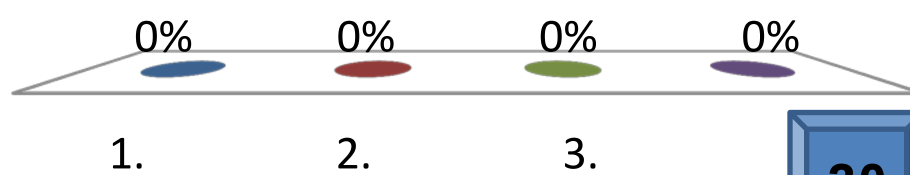
http://www.acoustics.salford.ac.uk/feschools/waves/wine3video.htm

Consider a tractor driving across a field that has undulations at regular intervals. The distance between the bumps is about 4.2 m. Because of safety reasons, the tractor does not have a suspension system but the driver's seat is attached to a spring to absorb some of the shock as the tractor moves over rough ground. The natural frequency of the driver and the seat is f = 2 Hz. At what speed does the drive become dangerous?



Hint: What is v when the driver hits a bump every T = 1/f second?

1. 2.1 m/s = 4.7 mph
2. 4.2 m/s = 9.4 mph
3. 8.4 m/s = 18.8 mph
4. 12.6 m/s = 28.2.8 mph



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The distance between the bumps is about 4.2 m. The natural frequency of the driver and the seat is $f = 2$ Hz. At what speed does the drive become dangerous?

The drive becomes dangerous when the frequency with which the tractor encounters the bumps equals the natural frequency of the driver and the seat, or when the time interval between hitting the bumps equals the natural period.

$f = 2$ Hz = 2/s, $T = 0.5$ s.

$v = (4.2\text{ m})/T = 8.4$ m/s.

Traveling waves

Periodic, sinusoidal waves in one dimension:
The displacement as a function of position and time is given by

$$y(x,t) = A \sin(kx - \omega t + \phi)$$

for a wave traveling in the +x direction, and by

$$y(x,t) = A \sin(kx + \omega t + \phi)$$

for a wave traveling in the -x direction.

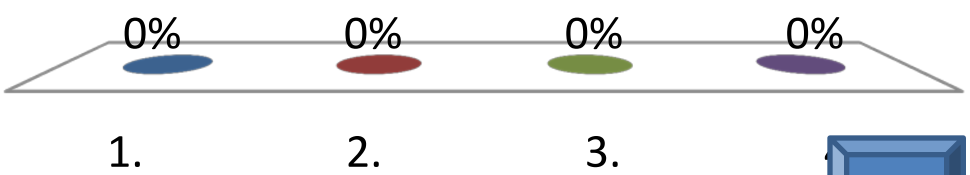
k is the wavenumber, $k = 2\pi/\lambda$.
$\omega = 2\pi/T = 2\pi f$ is the angular frequency.
ϕ is called the phase constant.
λ = wavelength, f = frequency, T = period, $v = \lambda/T = \lambda f = \omega/k$ = speed.
A is the amplitude.

The energy E transported by the wave is proportional to A^2.
The power P is proportional to $A^2 v$.

For waves on a string: $v = (F/\mu)^{1/2}$

A wave travelling in the +x direction is described by the equation
$y = 0.1 \sin(10x - 100t)$,
where x and y are in meters and t is in seconds. Calculate
(i) the wavelength,
(ii) the period,
(iii) the speed, and
(iv) the amplitude of the wave

1. (i) 10 m, (ii) 0.100 s, (iii) 1 m/s, (iv) 0.1 m
2. (i) 0.63 m, (ii) 15.9 s, (iii) 10 m/s, (iv) 0.2 m
3. (i) 0.1 m, (ii) 0.01 s, (iii) 10 m/s, (iv) 0.1 m
4. (i) 0.63 m, (ii) 0.063 s, (iii) 10 m/s, (iv) 0.1 m



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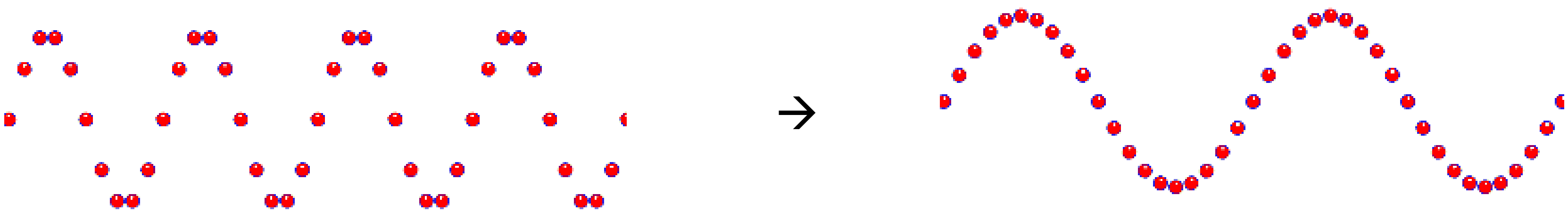
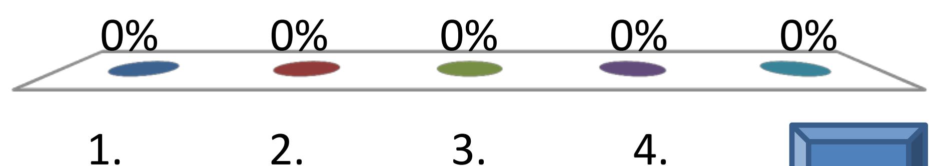
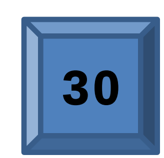
If you double the wavelength λ of a wave on a string under fixed tension, what happens to the wave speed v and the wave frequency f?



1. v is doubled and f is doubled.
2. v is doubled and f is unchanged.
3. v is unchanged and f is halved.
4. v is unchanged and f is doubled.
5. v is halved and f is unchanged.





Which of the following wave functions describe a wave that moves in the -x-direction?

1. $y(x,t) = A \sin(-kx - \omega t)$
2. $y(x,t) = A \sin(kx + \omega t)$
3. $y(x,t) = A \cos(kx + \omega t)$

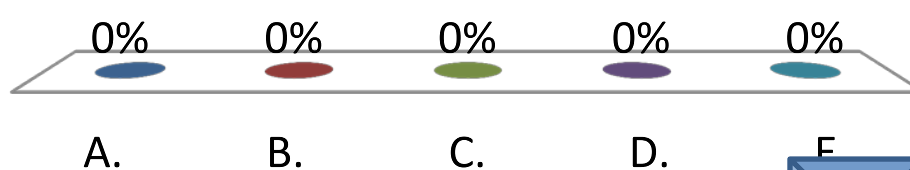
A. 1 only

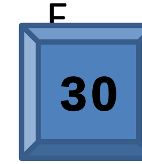
B. 2 only

C. 3 only

D. 2 and 3 only

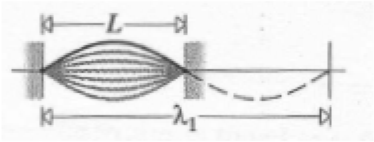
E. 1, 2, and 3

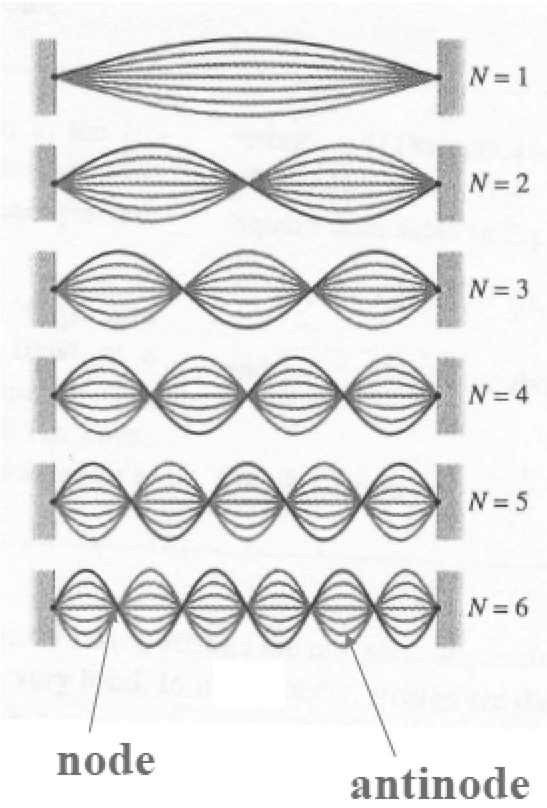




Standing waves on a string

For a string **fixed on both ends** a **standing wave** forms when an integral number of half wavelengths fit into the length of the string.





Fundamental:

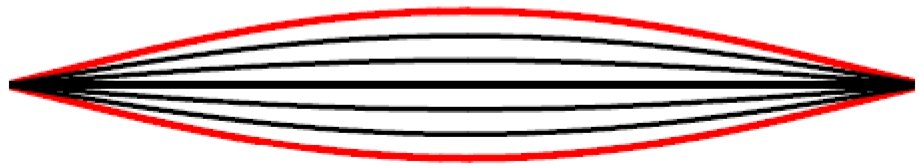
$$f_1 = \frac{v}{2L}$$

Resonances:

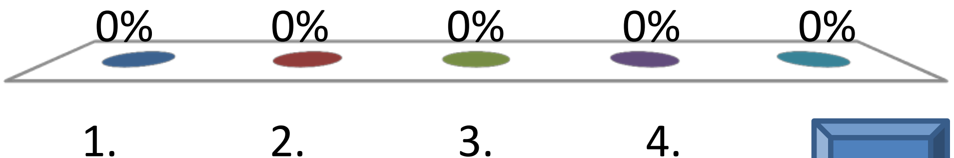
$$f_N = N f_1$$

$$v = \sqrt{\frac{F}{\mu}}$$

A string of length 100 cm is held fixed at both ends and vibrates in a standing wave pattern. The first harmonic is shown. The wavelengths of the standing waves making up the pattern **cannot** be



1. 400 cm.
2. 200 cm.
3. 100 cm.
4. 66.7 cm.
5. 50 cm



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