Physics 221, February 18

Key Concepts:

• Definition of momentum and impulse
• Conservation of momentum
• The center of mass
• Rockets
Please do not use social media during class.
Linear momentum

Momentum: \( p = mv \) \hspace{1cm} \text{(vector)}

Rate of change: \( \frac{\Delta p}{\Delta t} = m\frac{\Delta v}{\Delta t} = ma = F \) \hspace{1cm} \text{(vector)}

\[ F_x = \frac{\Delta p_x}{\Delta t}, \quad F_y = \frac{\Delta p_y}{\Delta t} \]

Impulse: \( I = \Delta p = p_f - p_i = F\Delta t \) \hspace{1cm} \text{(vector)}

Kinetic energy: \( E_{\text{kin}} = \frac{1}{2}mv^2 = \frac{p^2}{2m} \) \hspace{1cm} \text{(scalar)}

\[ p = \left(2mE_{\text{kin}}\right)^{1/2} \] \hspace{1cm} \text{(magnitude)}
When an increase in speed triples the kinetic energy of a moving object, the magnitude of its momentum 

1. increases by a factor of √3.
2. triples.
3. increases by a factor of 9.
4. decreases by a factor of 3.
5. can increase or decrease depending on factors not stated.
Two objects have masses $m_1$ and $m_2$, respectively. If $m_2 = 4m_1$, and both have the same kinetic energy, which has more momentum (magnitude)?

For example:

1. Object 1 with mass $m_1$.
2. Object 2 with mass $m_2$.
3. The magnitudes of their momenta are the same.

The same amount of elastic potential energy was converted to kinetic energy.

The same net work was done.
A piece of clay with mass $m = 0.01$ kg collides with the floor at speed of $8$ m/s and sticks. The collision takes $0.02$ s. The magnitude of the average force the piece of clay experiences during the collision is

1. 0 N.
2. 1 N.
3. 2 N.
4. **4 N.**
5. 8 N.

Hint: $\Delta p = F \Delta t$
You have heard the tried and true phrase “it is like running in to a brick wall”. Now it is time to dig a little deeper and modify the phrase.

Assume you are in a car driving and come in contact with this proverbial brick wall. Your car can do three things after the strike: it can go through, come to stop or bounce back. Select the option which will be the most dangerous to you assuming similar collision times.

1. Going through the wall.
2. Coming to a stop.
3. **Bouncing back.**
4. All options are equally dangerous.
A ball (mass 0.40 kg) is initially moving to the left at 30 m/s. After hitting the wall, the ball is moving to the right at 20 m/s. What is the impulse of the ball receives during its collision with the wall?

1. 20 kg m/s to the right
2. 20 kg m/s to the left
3. 4 kg m/s to the right
4. 4 kg m/s to the left
5. None of the above
Conservation of momentum

For a system of objects, a component of the momentum along a chosen direction is constant, if no net outside force with a component in this chosen direction acts on the system.

In collisions between isolated objects momentum is always conserved.
\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

Kinetic energy is only conserved in elastic collisions.
\[ \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 \]

In explosions or disintegrations momentum is conserved.
\[ (\Sigma m_i v_i)_{\text{before}} = (\Sigma m_i v_i)_{\text{after}} \]

Kinetic energy is not conserved.
Stored potential energy is converted into ordered or disordered kinetic energy.
Two balls of putty of equal mass approach each other from opposite directions with equal speeds. They stick together and come to rest. Was momentum conserved in this collision?

1. Yes
2. No
3. Maybe, we need more information.
You have a mass of 60 kg. You are standing on an icy pond, when your “friend” throws a 10 kg ball at you with horizontal velocity of 7 m/s.

If you catch the ball, how fast will you be moving?

1. 0 m/s
2. 1 m/s
3. 1.17 m/s
4. 3.5 m/s
5. 7 m/s
Demonstrations

Collisions and conservation of momentum

Newton’s cradle
http://www.youtube.com/watch?v=mFNe_pFZrsA

Astroblaster
http://www.youtube.com/watch?v=cloY0R5mj2s&feature=related
Center of mass

- The center of mass (CM) of a system moves as if the total mass of the system were concentrated at this special point.
- It responds to external forces as if the total mass of the system were concentrated at this point.
- The total momentum of the system only changes, if external forces are acting on the system.
- The center of mass of the system only accelerates, if external forces are acting on the system.
- Coordinates of the center of mass (CM):

\[
x_{\text{CM}} = \frac{\sum m_i x_i}{M}, \quad y_{\text{CM}} = \frac{\sum m_i y_i}{M}, \quad z_{\text{CM}} = \frac{\sum m_i z_i}{M}
\]

\[
M = \sum m_i
\]
Two particles of masses 2 g and 8 g are separated by a distance of 6 cm. The distance of their center of mass from the heavier particle is

1. 1.5 cm
2. **1.2 cm**
3. 3 cm
4. 4.8 cm
5. 2 cm
A baseball bat with uniform density is cut at the location of its center of mass as shown in the figure. The piece with the smaller mass after the cut is

1. the piece on the left.
2. the piece on the right.
3. Both pieces have the same mass.
4. This is impossible to determine.
A radioactive nucleus of mass $M$ moving along the positive $x$-direction with speed $v$ emits an $\alpha$-particle of mass $m$. If the $\alpha$-particle proceeds along the negative $x$-direction, the centre of mass of the system (made of the daughter nucleus and the $\alpha$-particle) will

1. remain at rest.
2. move along the positive $x$-direction with speed less than $v$.
3. move along the positive $x$-direction with speed greater than $v$.
4. move in a direction inclined to the positive $x$-direction.
5. move along the positive $x$-direction with speed equal to $v$. 
The rocket principle

System consisting of many parts:
no external force $\iff$ no acceleration of the CM

But different parts of the system can accelerate with respect to the CM, as long as the total momentum of the system is constant.

Examples:

http://www.youtube.com/watch?v=D-5TovPg4F4
Suppose you are on a cart, initially at rest on a frictionless, horizontal track. You throw a series of identical balls against a wall that is rigidly mounted to the cart. If the balls are thrown at a steady rate and bounce straight back, is the cart put into motion?

1. Yes, it starts to move to the right with constant speed.
2. Yes, it starts to move to the right and steadily gains speed.
3. Yes, it starts to move to the left with constant speed.
4. Yes, it starts to move to the left and steadily gains speed.
5. No, it remains in place.
A 55 kg physics student is at rest on a 5 kg sled that also holds a chunk of ice with a mass of 1.5 kg. The student throws the ice horizontally with a speed of 12 m/s relative to the ground. If the sled slides over a frozen pond without friction, how fast (in m/s) are the sled and student traveling with respect to the ground after throwing the chunk of ice?

1. 0.3 m/s
2. 0.327 m/s
3. 5 m/s
4. 0.2 m/s
5. 0.218 m/s