

Physics 221, February 23

Key Concepts:

- Rotations
- Kinematics and dynamics
- Energy and angular momentum
- Equilibrium

Please do not use social media during class.

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**“In lieu of a raise or promotion,
I will agree to friend you on Facebook.”**

When we describe how the **linear motion** of an object changes when forces are acting on it, we describe how the **motion of the center of mass (CM)** of the object changes.

But what about the **motion about the CM**? How do we describe rotations, rolling, etc?

Rotations

Extended object can have **translational** and **rotational** motion.

How do we describe rotational motion?

Angular speed: $\omega = \Delta\theta/\Delta t$

Angular velocity: $\boldsymbol{\omega} = \Delta\theta/\Delta t \mathbf{n}$,
 \mathbf{n} = direction indicator

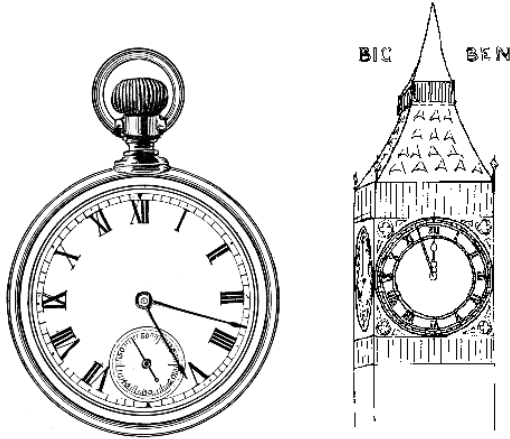
Angular acceleration: $\boldsymbol{\alpha} = \Delta\boldsymbol{\omega}/\Delta t$

Right-hand rule: direction of angular velocity

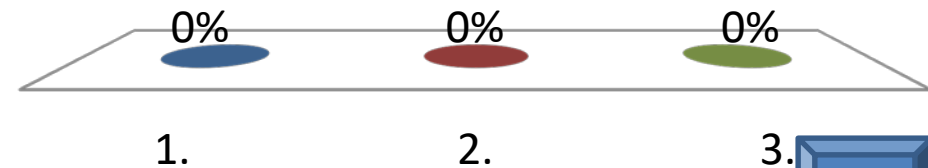
The direction of the **angular acceleration** is the direction of the **change** of the angular velocity.



A pocket watch and Big Ben are both keeping perfect time.
Which minute hand has the larger angular speed ω ?

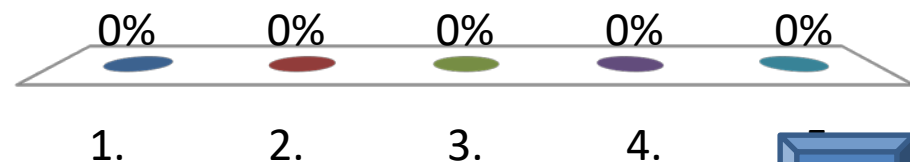


1. Big Ben's
2. The pocket watch's
3. They both have the same ω .



Angular speed is measured in units of s^{-1} , which is rad/s.
How many degrees correspond to 1 radian?

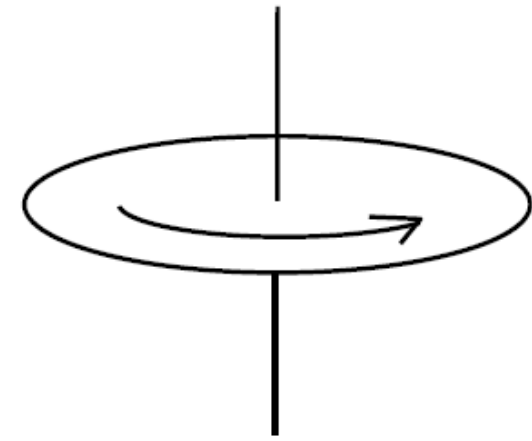
1. 1 rad = 2π degrees
2. 1 rad = 180 degrees
3. 1 rad = 10 degrees
4. 1 rad = 57.3 degrees
5. The question makes no sense.



A disk is spinning with angular velocity ω as shown. It begins to slow down. While it is slowing, what are the

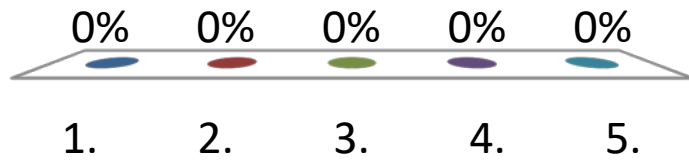
(i) directions of its angular velocity ω

(ii) and its angular acceleration α ?



A) \rightarrow B) \leftarrow C) \uparrow D) \downarrow

1. A, C
2. A, B
3. C, C
4. **C, D**
5. Some other directions.



Hint
 Angular acceleration does not necessarily point in the direction of the angular velocity.
 For a fixed rotation axis, it points in the direction of ω if the rotation rate is increasing and opposite to the direction of ω if the rotation rate is decreasing.

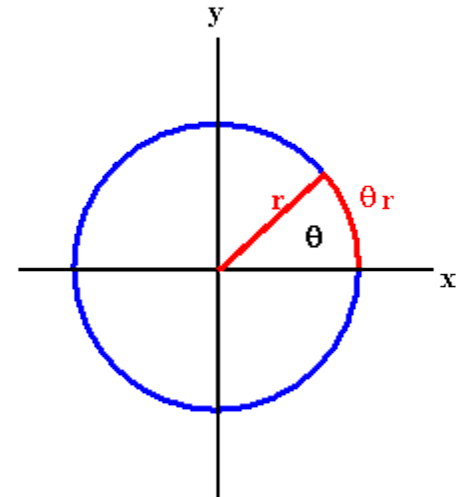
Speed and angular speed

When a wheel rotates about the z-axis, each point on the wheel has the same **angular speed ω** .

The **linear speed v** of a point P, however, depends on its distance from the axis of rotation.

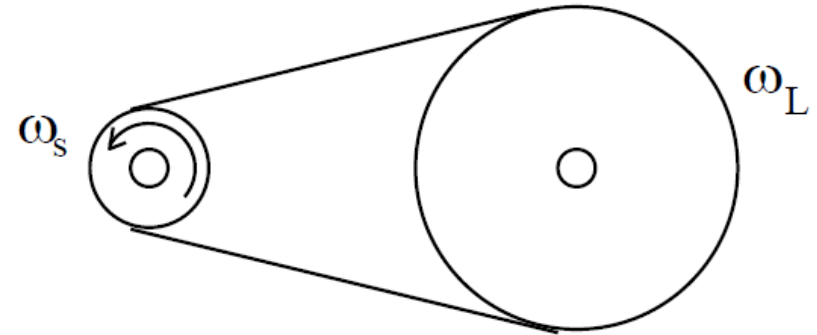
In terms of the angular speed ω , the speed v of the point P is **$v = \omega r$** .

v is the tangential velocity of the point P.



A small wheel and a large wheel are connected by a belt. The small wheel is turned at a constant angular speed ω_s .

How does the angular speed of the large wheel ω_L compare to that of the small wheel?

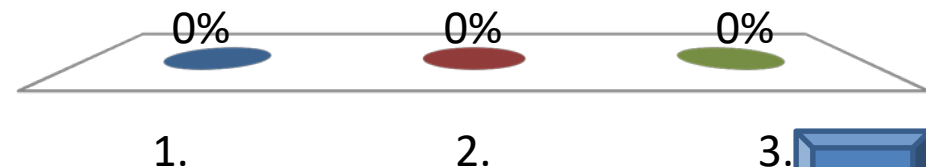


Hint:

The belt has the same linear speed everywhere.

$$\omega = v/r$$

1. $\omega_s = \omega_L$
2. $\omega_s > \omega_L$
3. $\omega_s < \omega_L$



Kinematics and dynamics

Motion with **constant angular acceleration**

Kinematic equations: $\omega_f = \omega_i + \alpha(t_f - t_i)$

$$\theta_f = \theta_i + \omega_i(t_f - t_i) + \frac{1}{2} \alpha(t_f - t_i)^2$$

$$\omega_{\text{avg}} = (\omega_i + \omega_f)/2.$$

$$\omega_f^2 = \omega_i^2 + 2 \alpha(\theta_f - \theta_i).$$

A fan rotating with an initial angular velocity of **1000 rev/min** is switched off. In **2 seconds**, the angular velocity decreases to **200 rev/min**. Assuming the angular acceleration is constant, how many revolutions does the blade undergo during this time?

Given:

$$\omega_i = 1000 \text{ rev/min} \quad \omega_f = 200 \text{ rev/min}, \quad \Delta\omega = -800 \text{ rev/min}$$

$$\alpha = \text{constant}, \quad \Delta t = 2 \text{ s}$$

Asked:

$$\Delta\theta = ?$$

Possible approach to the solution:

$$\text{Find } \omega_{\text{avg}} = (\omega_i + \omega_f)/2, \text{ then use } \Delta\theta = \omega_{\text{avg}}\Delta t.$$

Another approach to the solution:

$$\text{Find } \alpha = (\omega_f - \omega_i) / \Delta t, \text{ then use } \Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha\Delta t^2.$$

A student sees the following question on an exam:

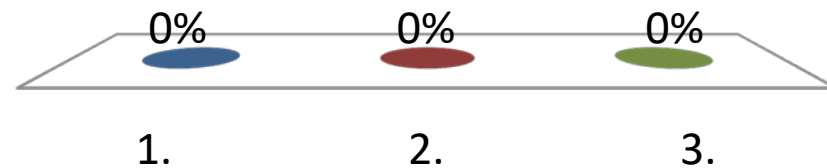
A flywheel with mass 120 kg and radius 0.6 m, **starting at rest**, has an **angular acceleration** of 0.1 rad/s^2 . How many revolutions has the wheel undergone after **10 s**?

Which single formula might the student use to answer the question?

1. $\omega_f = \omega_i + \alpha(t_f - t_i)$

2. $\theta_f = \theta_i + \omega_i(t_f - t_i) + \frac{1}{2} \alpha(t_f - t_i)^2$

3. $\omega_f^2 = \omega_i^2 + 2 \alpha(\theta_f - \theta_i)$



Cross product or vector product

Let $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ be the **cross product** of two vectors \mathbf{A} and \mathbf{B} . \mathbf{C} is a vector.

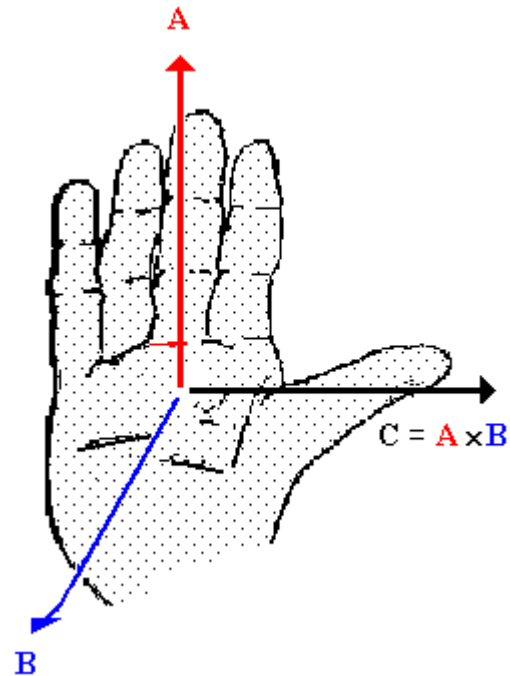
The **magnitude** of \mathbf{C} is $C = AB\sin\theta$, where θ is the smallest angle between the directions of the vectors \mathbf{A} and \mathbf{B} .

The **direction** of \mathbf{C} can be found using the **right-hand rule**.

Point the fingers of your right hand in the direction of \mathbf{A} .

Orient the palm of your hand so that, as you curl your fingers, you can sweep them over to point in the direction of \mathbf{B} .

Your thumb points in the direction of $\mathbf{C} = \mathbf{A} \times \mathbf{B}$.



A net torque causes angular acceleration.

Angular acceleration: $\alpha = \tau / I$ (α and τ point in the same direction.)

Torque is a vector. It is the **cross product** of a lever arm \mathbf{r} and a force \mathbf{F} .

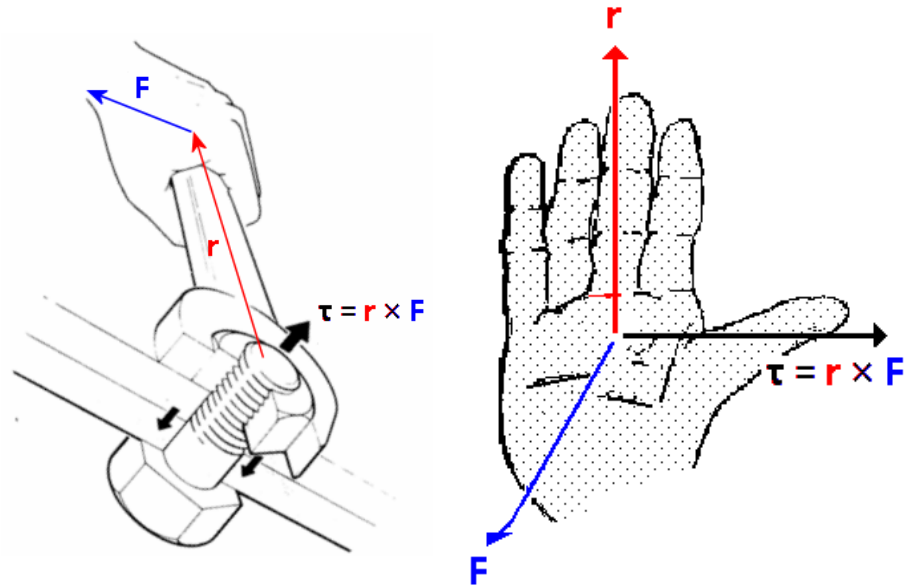
torque = lever arm \times force

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

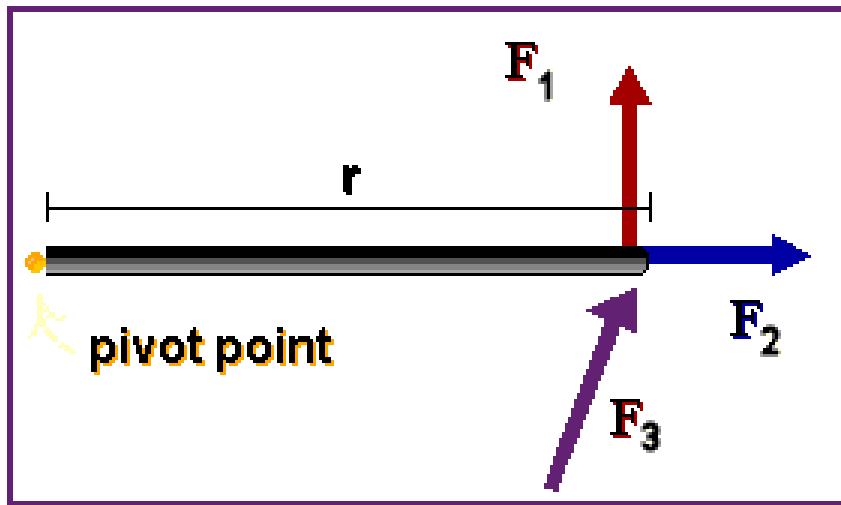
Direction of the torque:

Let the fingers of your right hand point from the axis of rotation to the point where the force is applied.

Curl them into the direction of \mathbf{F} .
Your thumb points in the direction of the torque vector.



F_1 , F_2 and F_3 have the same magnitude. Which statement is **correct**?



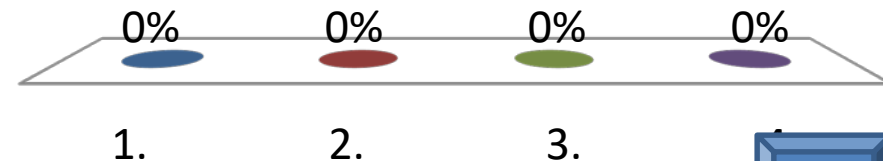
Hint:

magnitude: $\tau = r F \sin\theta$

r = vector pointing from axis of rotation to point where force F is applied.

θ = smallest angle between r and F .

1. All forces produce a torque pointing out of the page.
2. The torque produced by F_1 and F_3 point in opposite directions.
3. F_2 produces no torque, and the torques produced by F_1 and F_3 have equal magnitudes,
4. F_1 produces the torque with the largest magnitude.



Moment of inertia

$$\tau = I \alpha$$

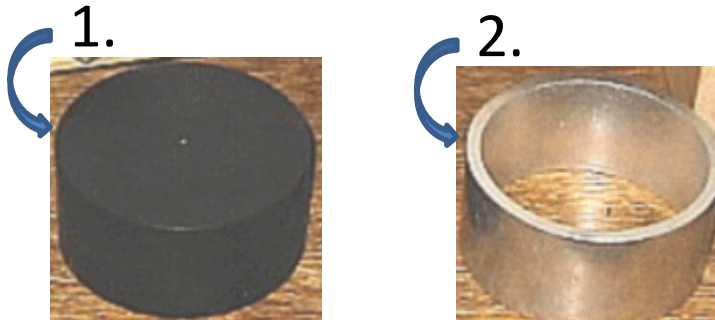
The moment of inertia I of an object is defined with respect to the axis of rotation.

$$I = \sum m_i r_i^2$$

The farther the bulk of the mass is from the axis of rotation, the greater is the moment of inertia of the object about the axis.

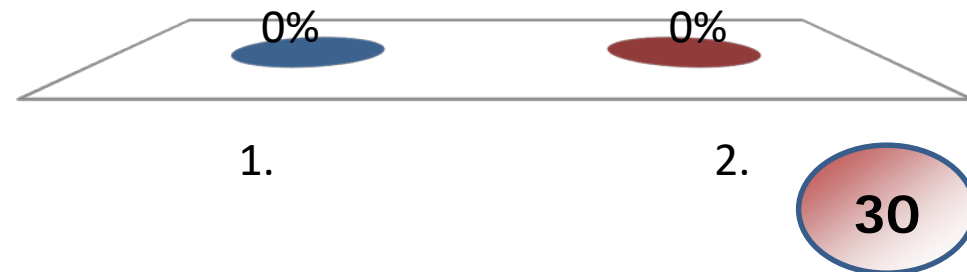
larger I \leftrightarrow larger τ needed to produce same angular acceleration

Both the disk and the ring have the **same mass**, and the **same radius**. Which has the larger moment of inertia about the center?



Hint:
The farther the bulk of the mass of an object is from an axis, the larger is the moment of inertia of the object about this axis.

1. The disk
2. **The ring**



Energy and angular momentum

Energy of an object with **translational** and **rotational** motion:

$$\text{translational kinetic energy} = \frac{1}{2}mv_{\text{CM}}^2$$

$$\text{rotational kinetic energy} = \frac{1}{2}I\omega^2$$

$$\text{total kinetic energy} = \frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2$$

Rolling: $v_{\text{CM}} = r\omega$

total kinetic energy $= \frac{1}{2} (m + I/r^2)v_{\text{CM}}^2$

Ratio $E_{\text{trans}}/E_{\text{rot}} = mr^2/I$

Angular momentum:

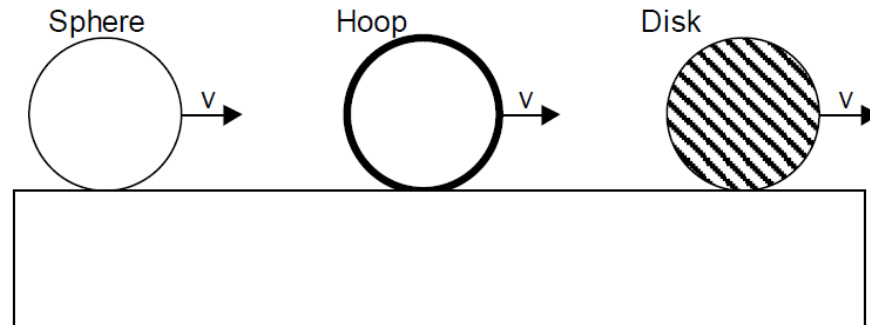
$$\mathbf{L} = I\boldsymbol{\omega}, \quad \Delta\mathbf{L}/\Delta t = I\boldsymbol{\alpha} = \boldsymbol{\tau}, \quad \Delta\mathbf{L} = \boldsymbol{\tau}\Delta t$$

If no external torque acts on a system of interacting objects, then the total angular momentum of the objects is constant.

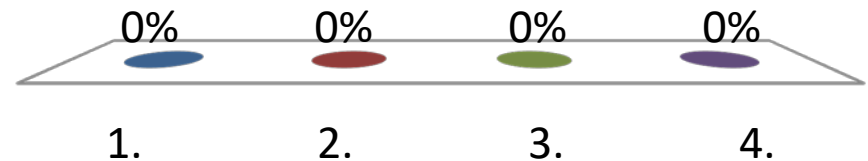
A sphere, a hoop, and a cylinder, all with the same mass M and the same radius R , are rolling along, all with the same speed v . Which has the most total kinetic energy?

Hint:

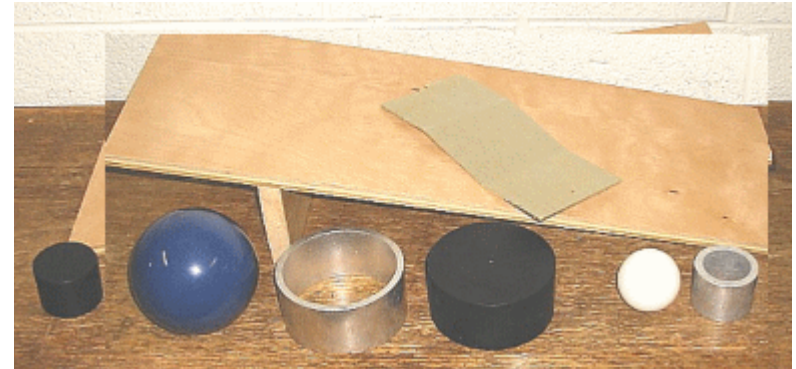
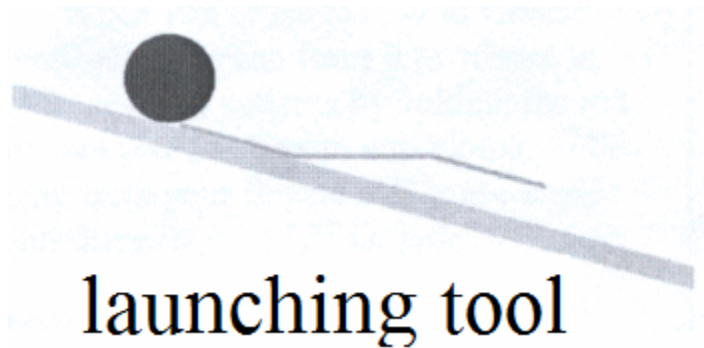
The farther the bulk of the mass of an object is from an axis, the larger is the moment of inertia of the object about this axis.



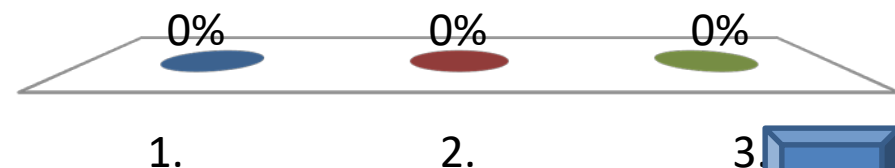
1. The hoop
2. The disk
3. The sphere
4. They all have the same kinetic energy.



Assume a disk and a ring with the same radius roll down an incline of height h and angle θ . If they both start from rest at $t = 0$, which one will reach the bottom first?



1. The disk
2. The ring
3. They both will arrive at the same time.



Some children are riding on the outside edge of a merry-go-round. Ignore friction in the rotation of the merry-go-round. Consider the "system" of the children plus the merry-go-round. At the same time, the children all move towards the center of the merry-go-round. When they do this



1. the **angular momentum** of the system stays constant.
2. the **moment of inertia** of the system stays constant.
3. the **angular velocity** of the system stays constant.
4. the merry-go-round slows down.

