Physics 221, February 22

Key Concepts:

- Rotations
- Kinematics and dynamics
- Energy and angular momentum
- Equilibrium

Center of mass

Newton's 2nd law, **F** = m**a**, when applied to an extended object, predicts the motion of a particular reference point for this object. This reference point is called the center of mass (CM).

The center of mass of a system responds to external forces as if the total mass of the system were concentrated at this point.

The coordinates of the center of mass (CM:

$$x_{CM} = \Sigma m_i x_i/M, \quad y_{CM} = \Sigma m_i y_i/M, \quad z_{CM} = \Sigma m_i z_i/M.$$

M is the total mass of the system.

$$M = \Sigma m_i.$$

When we describe how the linear motion of an object changes when forces are acting on it, we describe how the motion of the center of mass (CM) of the object changes.

But what about the motion about the CM? How do we describe rotations, rolling, etc?

Rotations

Extended object can have translational and rotational motion.

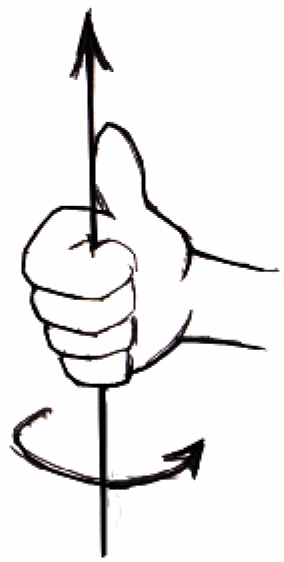
How do we describe rotational motion?

Angular speed: $\omega = \Delta\theta/\Delta t$

Angular velocity: $\boldsymbol{\omega} = \Delta\theta/\Delta t \ \mathbf{n}$,

$\mathbf{n}$ = direction indicator

Angular acceleration: $\boldsymbol{\alpha} = \Delta\boldsymbol{\omega}/\Delta t$

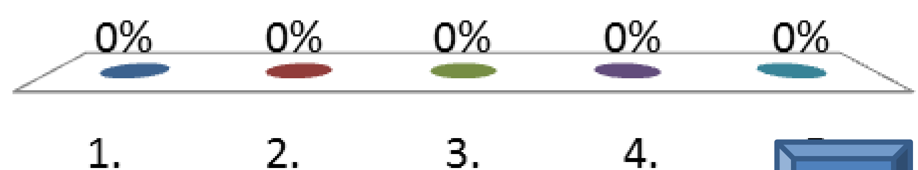


Right-hand rule: direction of angular velocity

The direction of the angular acceleration is the direction of the change of the angular velocity.

Angular speed is measured in units of s^{-1}, which is rad/s. How many degrees correspond to 1 radian?

1. 1 rad = 2π degrees
2. 1 rad = 180 degrees
3. 1 rad = 10 degrees
4. 1 rad = 57.3 degrees
5. The question makes no sense.



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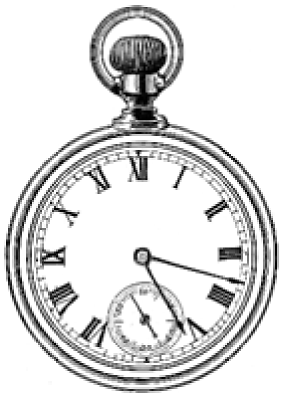
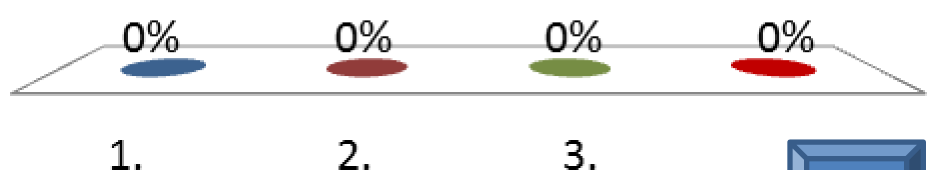
This pocket watch is keeping perfect time. What is the angular speed ω of its minute hand in units of rad/s?



1. 2π rad/s
2. 0.1 rad/s
3. 6 rad/s
4. $1.75*10^{-3}$ rad/s



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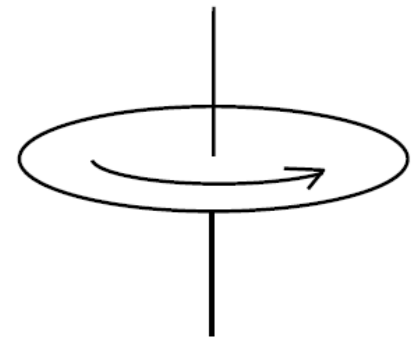
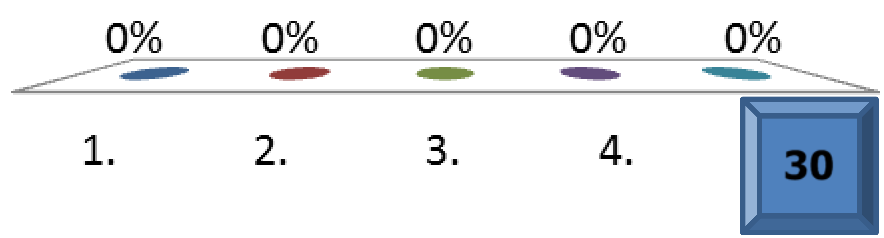
A disk is spinning with angular velocity **ω** as shown. It begins to speed up. While it is speeding up, what are the
(i) directions of its angular velocity **ω**
(ii) and its angular acceleration **α**?



A) → B) ← C) ↑ D) ↓

1. A, C
2. A, B
3. C, C
4. C, D
5. Some other directions.

Angular acceleration does not necessarily point in the direction of the angular velocity.
For a fixed rotation axis, it points in the direction of **ω** if the rotation rate is increasing and opposite to the direction of **ω** if the rotation rate is decreasing.



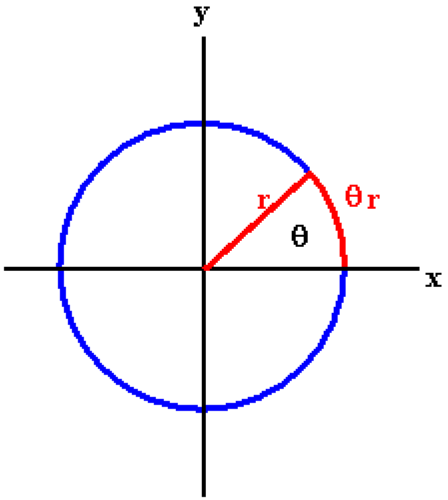
Speed and angular speed

When a wheel rotates about the z-axis, each point on the wheel has the same angular speed ω.

The linear speed v of a point P, however, depends on its distance from the axis of rotation.

In terms of the angular speed ω,
the speed v of the point P is $v = \omega r$.

v is the tangential velocity of the point P.



Kinematics and dynamics

Motion with constant angular acceleration

Kinematic equations: $\omega_f = \omega_i + \alpha(t_f - t_i)$

$$\theta_f = \theta_i + \omega_i(t_f - t_i) + ½\, \alpha(t_f - t_i)^2$$

$$\omega_{avg} = (\omega_i + \omega_f)/2.$$

$$\omega_f^{\,2} = \omega_i^{\,2} + 2\,\alpha(\theta_f - \theta_i).$$

A fan rotating with an initial angular velocity of 1000 rev/min is switched off. In 2 seconds, the angular velocity decreases to 200 rev/min. Assuming the angular acceleration is constant, how many revolutions does the blade undergo during this time?

Given:

ω_i = 1000 rev/min ω_f = 200 rev/min, $\Delta\omega$ = -800 rev/min

α = constant, Δt = 2s

Asked:

$\Delta\theta$ = ?

Possible approach to the solution:

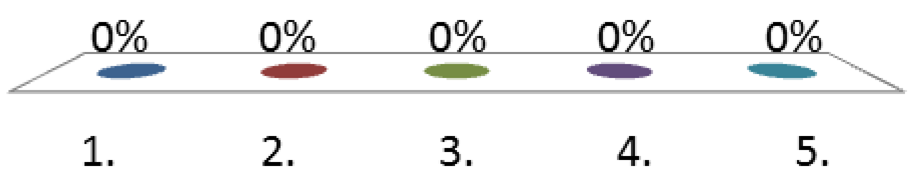
Find $\omega_{avg} = (\omega_i + \omega_f)/2$, then use $\Delta\theta = \omega_{avg}\Delta t$.

Another approach to the solution:

Find $\alpha = (\omega_f - \omega_i)/\Delta t$, then use $\Delta\theta = \omega_i\Delta t + ½\alpha\Delta t^2$.

A fan rotating with an initial angular velocity of 1000 rev/min is switched off. In 2 seconds, the angular velocity decreases to 200 rev/min. Assuming the angular acceleration is constant, how many revolutions does the blade undergo during this time?

1. 10
2. 20
3. 125
4. 100
5. 1200



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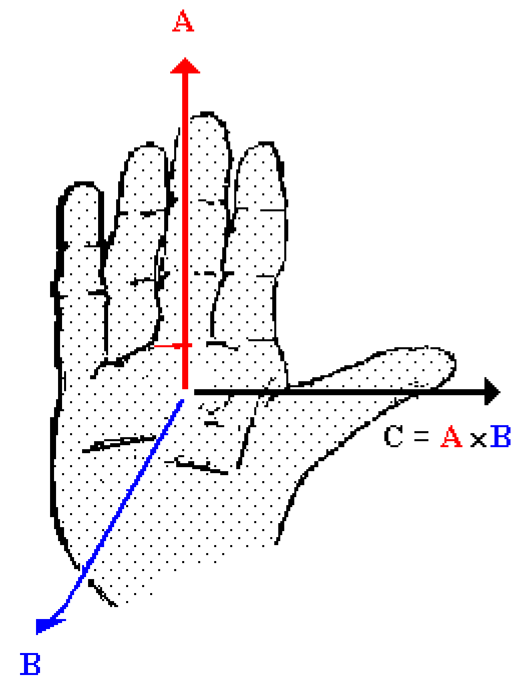
Cross product or vector product

Let **C** = **A**×**B** be the cross product of two vectors **A** and **B** . **C** is a vector.

The magnitude of **C** is C = ABsinθ,
where θ is the smallest angle between the directions of the vectors **A** and **B**.

The direction of **C** can be found using the **right-hand rule**.
Point the fingers of your right hand in the direction of **A**.
Orient the palm of your hand so that, as you curl you fingers, you can sweep them over to point in the direction of **B**.
Your thumb points in the direction of **C** = **A**×**B**.



A net torque causes angular acceleration.

Angular acceleration: $\boldsymbol{\alpha} = \boldsymbol{\tau} / I$ ($\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ point in the same direction.)

Torque is a vector. It is the **cross product** of a lever arm **r** and a force **F**.

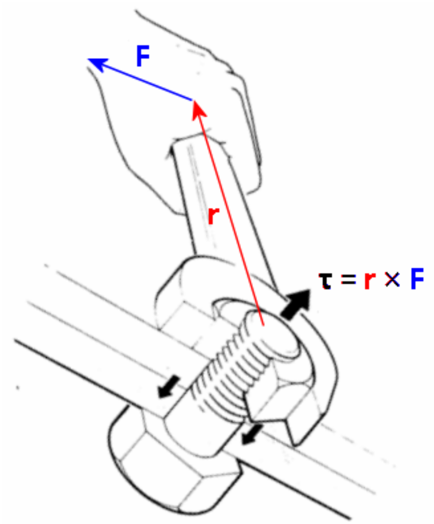
torque = lever arm × force

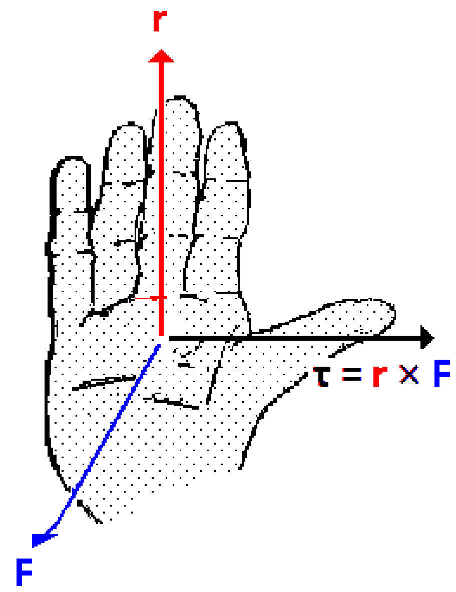
$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

Direction of the torque:

Let the fingers of your right hand point from the axis of rotation to the point where the force is applied.

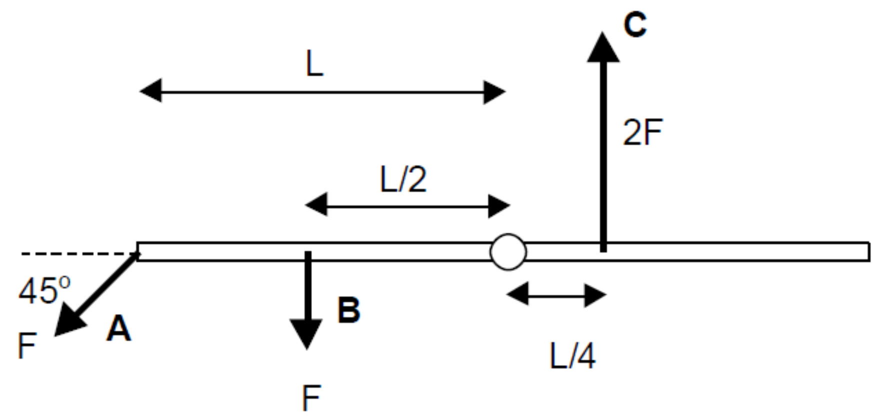
Curl them into the direction of **F**. Your thumb points in the direction of the torque vector.





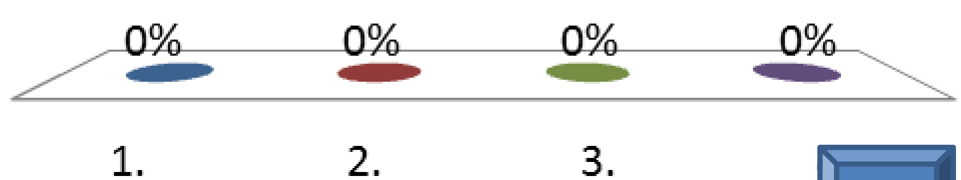
Three forces labeled A, B, C are applied to a rod which pivots on an axis through its center.

Which force produces the torque with the largest magnitude about this axis?



Hint:
magnitude: $\tau = r F \sin\theta$
r = vector pointing from axis of rotation to point where force **F** is applied.
θ = smallest angle between **r** and **F**.

1. A
2. B
3. C
4. All forces produce torques with equal magnitude.



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Moment of inertia

$$\tau = I\, \alpha$$

The moment of inertia I of an object is defined with respect to the axis of rotation.

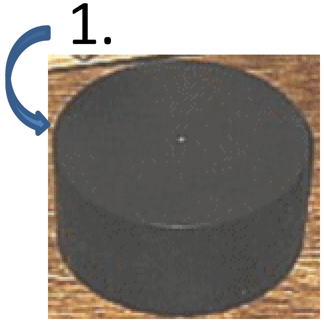
$$I = \sum m_i r_i^2$$

The farther the bulk of the mass is from the axis of rotation, the greater is the moment of inertia of the object about the axis.

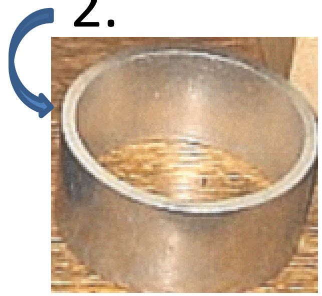
larger I ←→ larger τ needed to produce same angular acceleration

Both the disk and the ring have the same mass, and the same radius. Which has the larger moment of inertia about the center?

1.

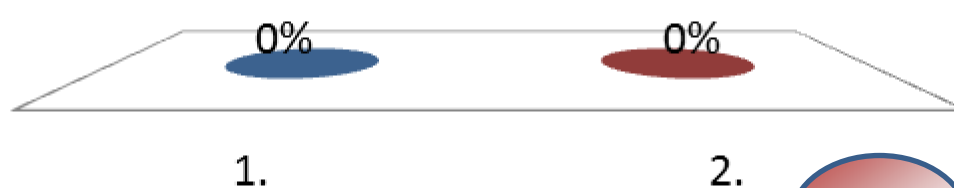


2.



Hint:
The farther the bulk of the mass of an object is from an axis, the larger is the moment of inertia of the object about this axis.

1. The disk
2. The ring



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Energy and angular momentum

Energy of an object with translational and rotational motion:

translational kinetic energy = $\frac{1}{2}mv_{CM}^2$

rotational kinetic energy = $\frac{1}{2}I\omega^2$

total kinetic energy = $\frac{1}{2}mv_{CM}^2 + \frac{1}{2}I\omega^2$

Rolling: $v_{CM} = r\omega$

total kinetic energy = $\frac{1}{2}(m + I/r^2)v_{CM}^2$

Ratio $E_{trans}/E_{rot} = mr^2/I$

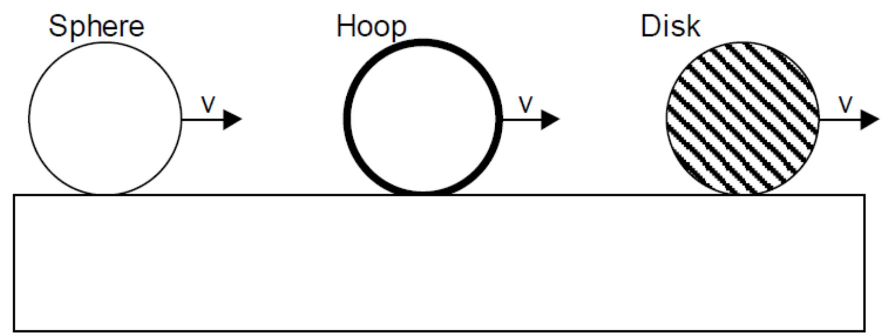
Angular momentum:

$\mathbf{L} = I\boldsymbol{\omega}$, $\Delta\mathbf{L}/\Delta t = I\boldsymbol{\alpha} = \boldsymbol{\tau}$, $\Delta\mathbf{L} = \boldsymbol{\tau}\Delta t$

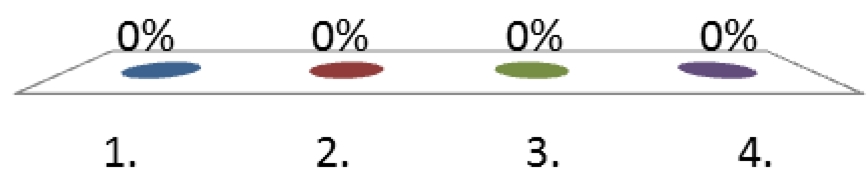
If no external torque acts on a system of interacting objects, then the total angular momentum of the objects is constant.

A sphere, a hoop, and a cylinder, all with the same mass M and the same radius R, are rolling along, all with the same speed v. Which has the most total kinetic energy?

Hint:
The farther the bulk of the mass of an object is from an axis,
the larger is the moment of inertia of the object about this axis.



1. The hoop
2. The disk
3. The sphere
4. They all have the same kinetic energy.



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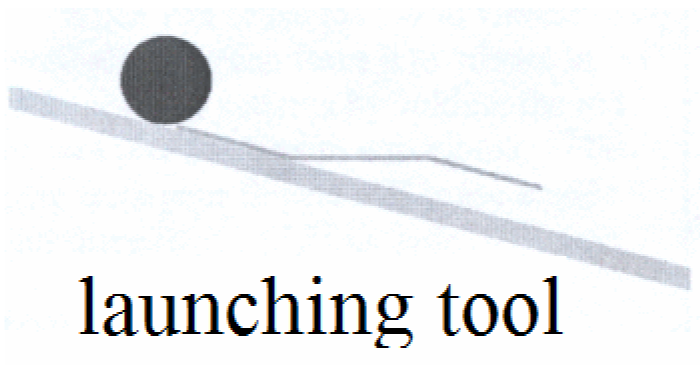
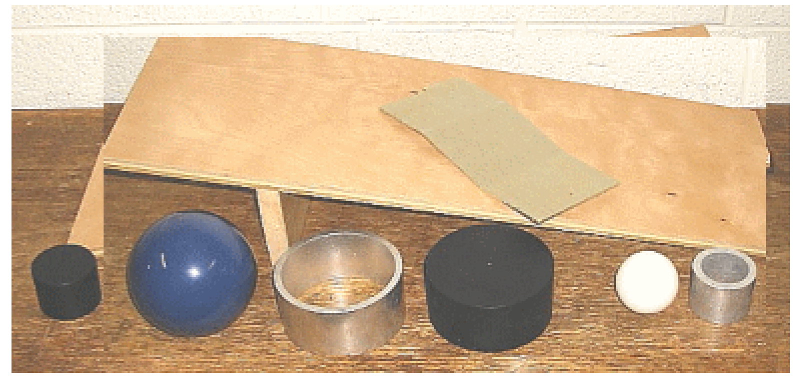
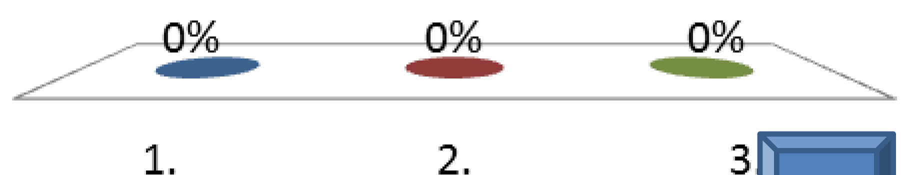
Assume a disk and a ring with the same radius roll down an incline of height h and angle theta. If they both start from rest at t = 0, which one will reach the bottom first?





1. The disk
2. The ring
3. They both will arrive at the same time.



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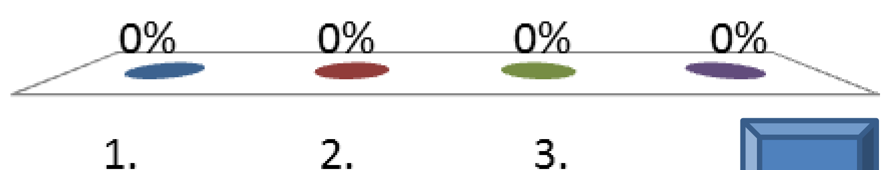
Some children are riding on the outside edge of a merry-go-round. Ignore friction in the rotation of the merry-go-round. Consider the "system" of the children plus the merry-go-round. At the same time, the children all move towards the center of the merry-go-round. When they do this

1. the angular momentum of the system stays constant.
2. the moment of inertia of the system stays constant.
3. the angular velocity of the system stays constant.
4. the merry-go-round slows down.



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