

Physics 221, March 9

Key Concepts:

- Dynamics of ideal fluids
- What does Bernoulli's equation mean?
- Viscosity
- Turbulence

Ideal Fluid Dynamics

Ideal fluids \leftrightarrow Incompressible fluids flowing without friction

equation of continuity: $(\text{Area } 1) \cdot v_1 = (\text{Area } 2) \cdot v_2$

Bernoulli's equation: $P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$

Consequences:

Hydrostatics:

$$P_{\text{bottom}} = P_{\text{top}} + \rho g (h_{\text{top}} - h_{\text{bottom}}) \quad \rightarrow$$

Pascal's law:

$P =$ same everywhere at the same height

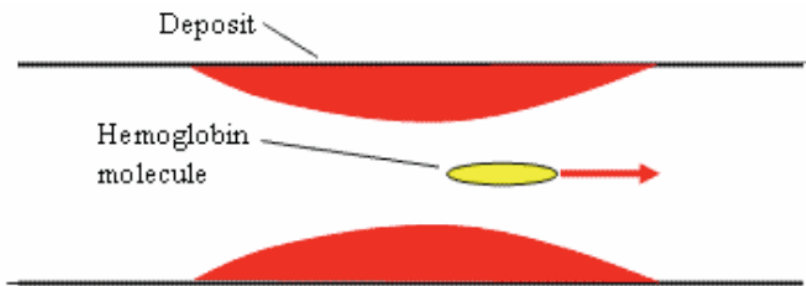
Flow in horizontal pipe:

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2 \quad \rightarrow$$

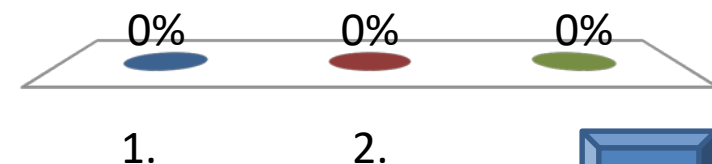
The faster the fluid is flowing, the lower is the pressure at the same height

Faster flow \rightarrow more KE in ordered motion \rightarrow less KE in random motion \rightarrow lower pressure

Blood flows through a section of a horizontal artery that is partially blocked by a deposit along the artery wall. A hemoglobin molecule moves **from the narrow region into the wider region**. What happens to the pressure acting on the molecule as it moves from the narrow into the wider region?



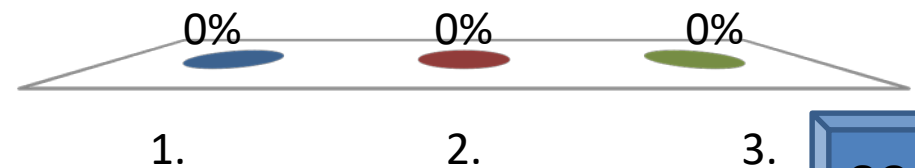
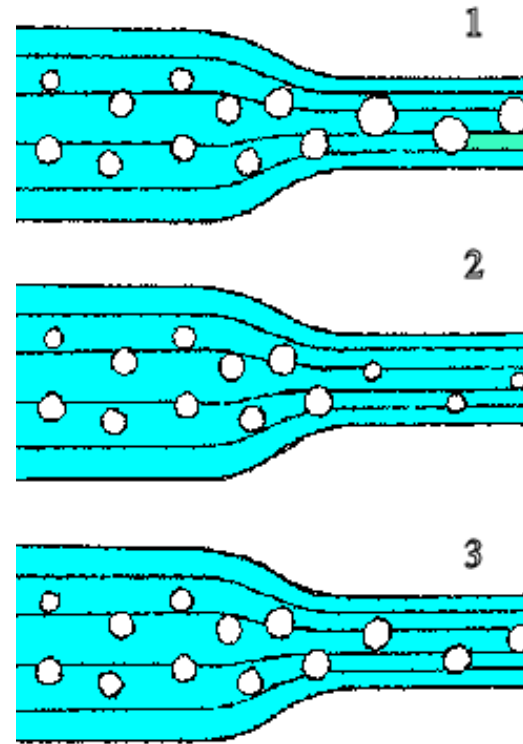
1. The pressure decreases.
2. There is no change in the pressure.
3. **The pressure increases.**



Water with air bubbles flows through a pipe that becomes narrower.

Compared to the wider region, the bubbles in the narrow region are

1. bigger.
2. smaller.
3. the same size.



Water distribution

The water level in the water tower changes very slowly, $v_1 \sim 0$.

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$$

→

$$P_1 + \rho g\Delta h = P_2 + \frac{1}{2}\rho v_2^2 = \text{constant}$$

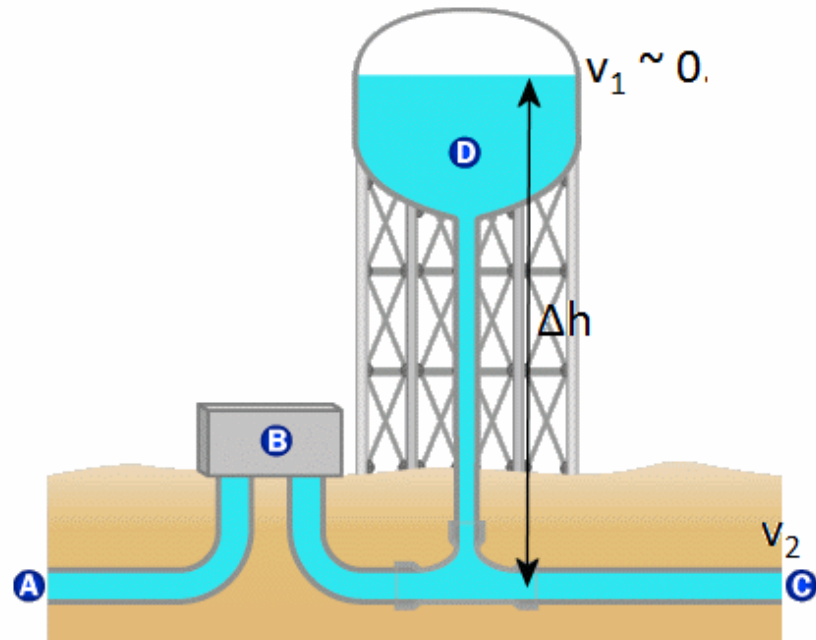
more water usage

→ higher water speed

in the supply pipe

→ lower water pressure

for the user



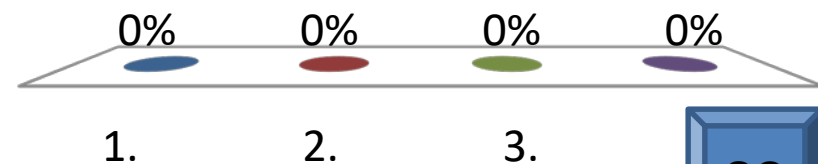
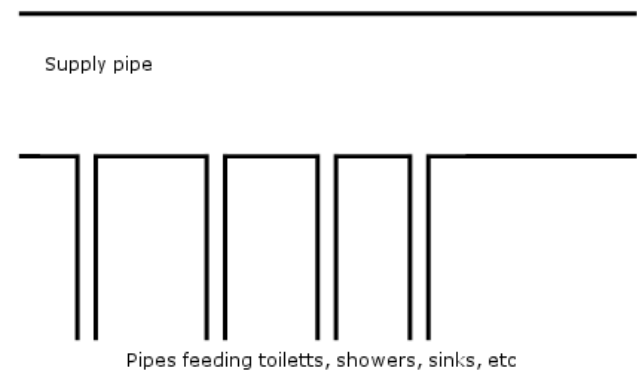
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A From the
treatment plant
B Pump

C To primary
feeders and customers
D Water

In your dorm room you watch the Vols play in an out-of-town game. It is a cliff hanger and the winner will be determined in the last few seconds of play. Everybody is taking their bathroom break right after the game ends. As the toilets flush and hands are washed, the faucet pressure is noticeably reduced, even though the supply pipe to the rooms can carry all the water needed. Why?

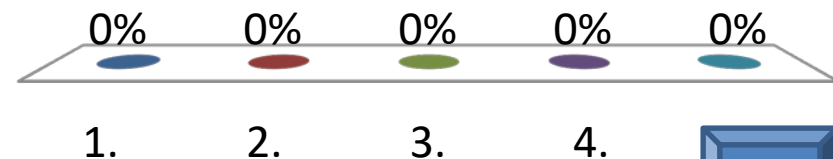
1. The water in the supply pipe has less gravitational potential energy than before the heavy usage.
2. More water is moving through the supply pipe and therefore the water has less kinetic energy, since energy is conserved.
3. The water has to speed up in the supply pipe and therefore the pressure drops in the pipe.
4. The water in the supply pipe slows down and therefore the pressure drops in the pipe.



Which one of the following is Bernoulli's equation not involved in explaining?



1. Why a roof can blow off a house in a hurricane.
2. The buoyant force on a floating iceberg.
3. Dynamic lift on airplane wings.
4. How fast water sprays out from a hole in a water tank.
5. It can explain all of these.



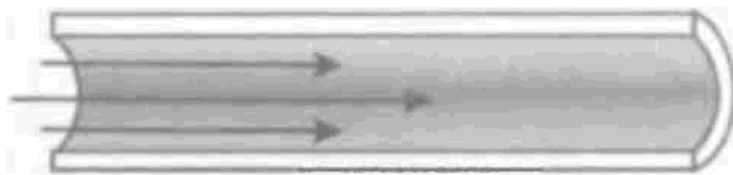
Viscosity (real fluids)

Ideal fluids flow without friction and do not dissipate ordered energy.

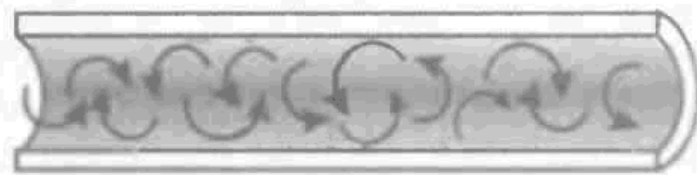
Real fluids convert ordered energy into disordered energy when flowing.

Viscosity is a measure of the internal friction of a flowing fluid. It is a result of intermolecular forces.

Ideal flow is always laminar. Viscous flow can be **laminar** or **turbulent**.



laminar flow

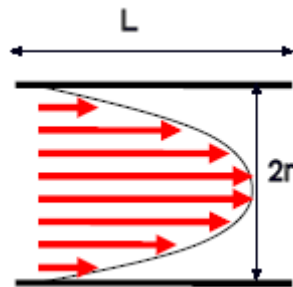


turbulent flow

Laminar viscous flow

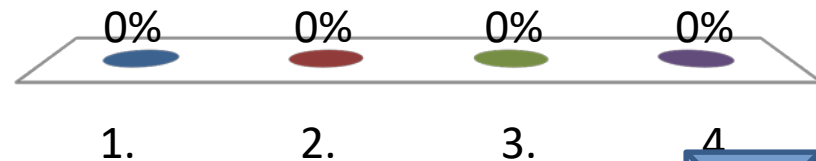
Characteristic:

A thin boundary layer of fluid next to a wall does not move at all.



When water flows smoothly in a straight hose, its viscosity causes the water to

1. speed up.
2. move with constant momentum.
3. move with constant velocity.
4. convert some of its kinetic energy into thermal energy.



How can you measure the viscosity η ?

Stokes' law:

When a sphere moves through a viscous fluid with constant velocity \mathbf{v} , the net force acting on it is zero. The applied force must cancel the viscous force, Whose magnitude is

$$F_{\text{visc}} = 6\pi\eta r_{\text{sphere}} \mathbf{v}.$$

(SI unit of η : Pa-s)

Let a sphere fall through a viscous fluid. Then the applied force is the sum of gravity and the buoyant force.

$$F_{\text{app}} = mg - B = (\rho_{\text{sphere}} - \rho_{\text{fluid}}) 4\pi (r_{\text{sphere}})^3 g / 3$$

Setting $F_{\text{app}} = F_{\text{visc}}$ yields

$$v_{\text{terminal}} = 2(\rho_{\text{sphere}} - \rho_{\text{fluid}})(r_{\text{sphere}})^2 g / (9 \eta).$$

Viscous drag

Consequences of viscosity:

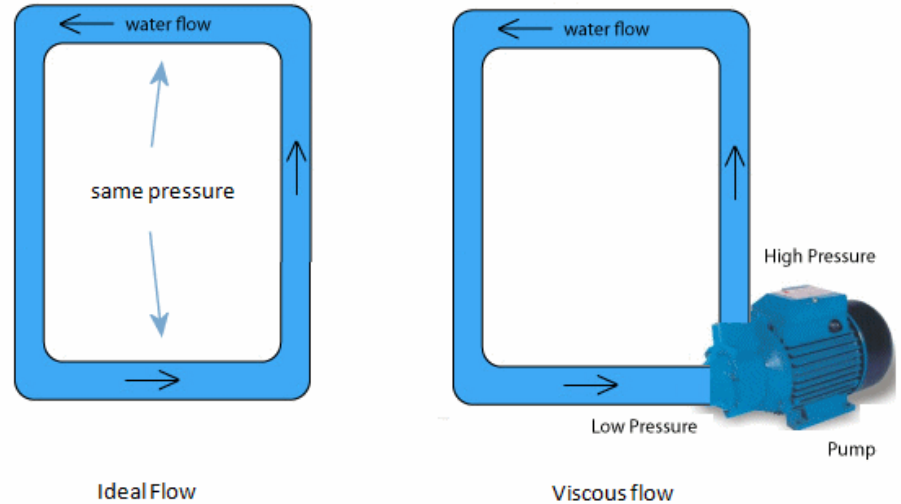
Poiseuille's law gives rate at which a viscous fluid flows through a hose or a pipe.

$$Q = \pi \Delta P r^4 / (8 \eta L)$$

Q = volume flow rate

ΔP = pressure difference across L of the pipe

r = radius of the pipe



Because of viscous drag, a pump needs to establish a pressure difference to keep the fluid flowing.

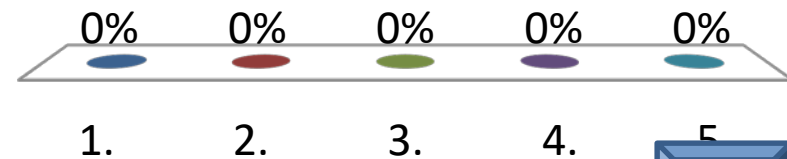
Suppose you work in the E.R. and a choking patient comes in with their trachea obstructed. The **diameter** is 1/3 of what it was before the obstruction. You manage to correct the problem and send them on their way safely, explaining that such obstructions are particularly dangerous because

1. the blocked airway passes only 1/3 the regular amount of air.
2. the blocked airway passes only 1/9 the regular amount of air.
3. the blocked airway passes only 2/3 the regular amount of air.
4. the blocked airway passes only 1/27 the regular amount of air.
5. The blocked airway passes only 1/81 the regular amount of air.

Hint:

$$Q = \frac{\pi \Delta P r^4}{8 \eta L}$$

$$Q \propto r^4$$



Why can't you get all the dust off your car by just squirting water from a hose onto it?

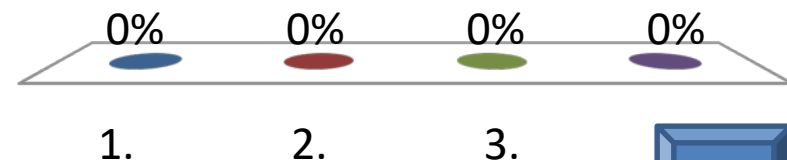
Why can't you simply remove dust just by blowing across a surface?

Why does dust cling to a fast rotating fan?

How can a leaf stay on a car moving at high speed?

What is one common answer to all these questions?

1. This is a consequence of **Bernoulli's equation.**
2. This is a consequence of the formation of stationary **boundary layer.**
3. This is a consequence of **Poiseuille's law.**
4. This is a consequence of **Stokes' law.**



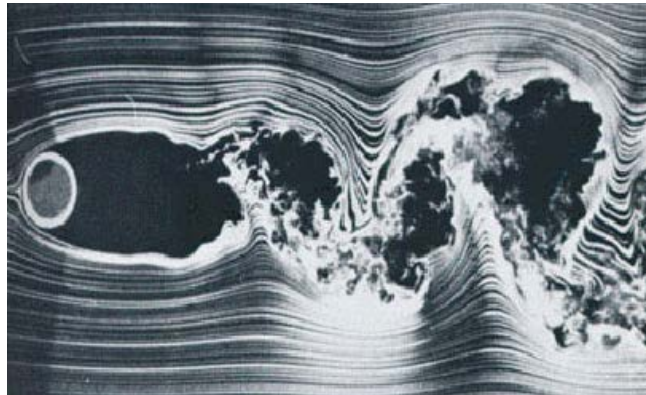
Turbulent flow

As the flow rate increases, a moving fluid can no longer sustain laminar flow. Fluid density, flow speed, viscosity, and geometry of the flow determine when viscous flow transitions from laminar to turbulent flow. A single number, called the **Reynolds number**, can be computed for any viscous flow.

$$R = \rho_{\text{fluid}} Dv/\eta$$

Here ρ_{fluid} is the **density** of the fluid, v is its **flow speed**, η is its **viscosity**, and D is a **characteristic length** that depends on the geometry.

When the Reynolds number **R becomes greater than ~ 2300** , the flow becomes turbulent.



Turbulence

Consequences of turbulence:

Ordered energy is “wasted” (converted into disordered energy.)
Bernoulli’s equation is not valid.

Poiseuille’s law is not valid, (Q is not proportional to ΔP .)
The equation of continuity still valid for incompressible fluids.

For flow through a smooth cylindrical pipe with inner diameter D



$$R = \rho_{\text{fluid}} Dv/\eta = \rho_{\text{fluid}} DvA/(\eta A) = \rho_{\text{fluid}} DQ/(\eta \pi D^2/4) = 4 \rho_{\text{fluid}} Q/(\eta \pi D) \propto Q/D.$$

The **Reynolds number**, is proportional to the volume flow rate and inverse proportional to the diameter.

A water pump generates a **constant flow Q** , which is causing a turbulent spray (Reynolds number $R = 3200$) out of this cylindrical nozzle:

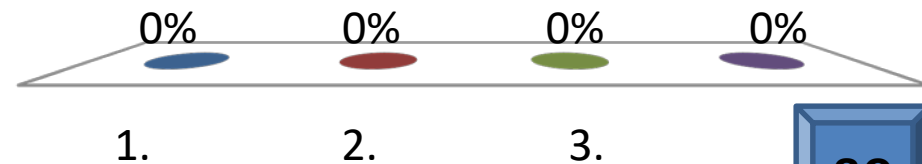


Hint:
 $R \propto Q/D$

Which of the following three nozzles could be substituted to achieve a streamline flow ($R < 2000$)?



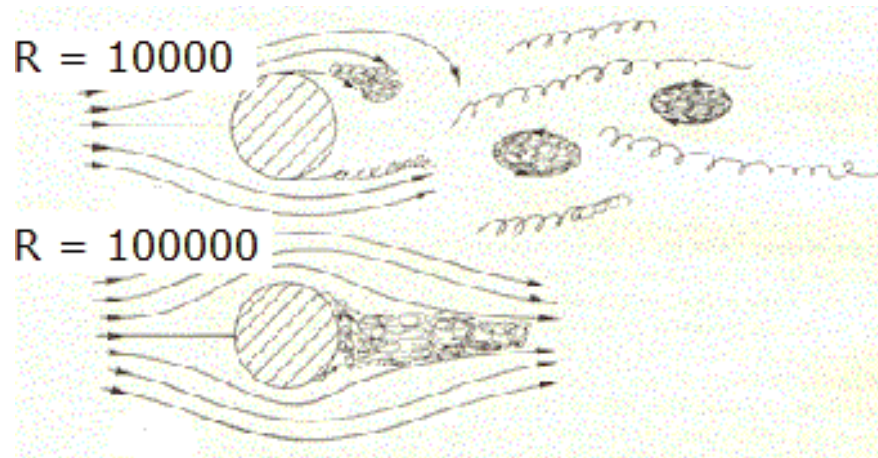
1. Tube I
2. Tube II
3. Tube III
4. **Tube I and III**



Viscous and pressure drag

- Viscosity produces **viscous drag**, even for laminar flow.
- Turbulence produces **pressure drag**.

Can we reduce pressure drag?

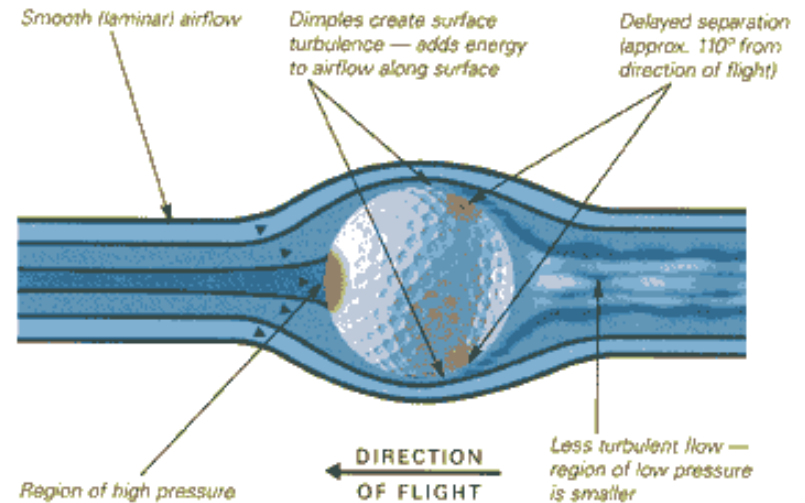
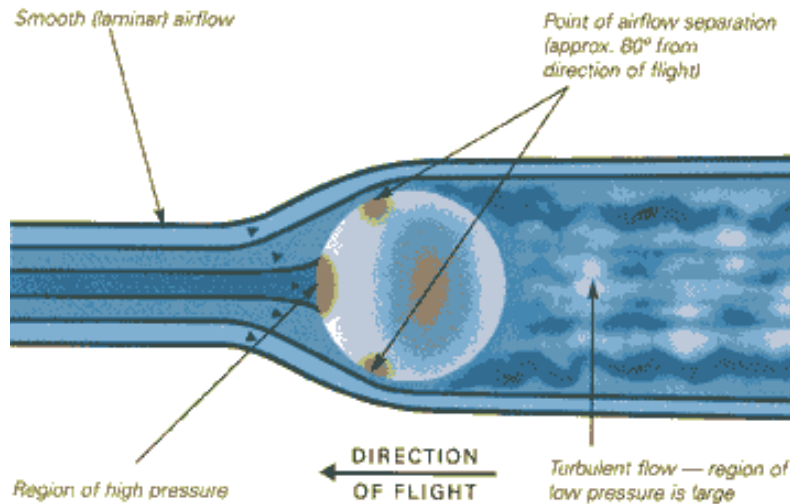


Yes, if R becomes greater than $\sim 10^5$, the flow again becomes semi-laminar.

The condition of the surface determines if R becomes greater than 100000 and a turbulent boundary layer forms at event moderate speeds. For rough surface R becomes greater than $\sim 10^5$ at lower speeds.

For semi-laminar flow Bernoulli's equation again becomes a valid approximation.

Compare the flight of two golf balls given the same impulse by the club. Which ball will travel a larger distance?



1. The smooth ball
2. The dimpled ball
3. They will travel the same distance since they have the same initial velocity.

