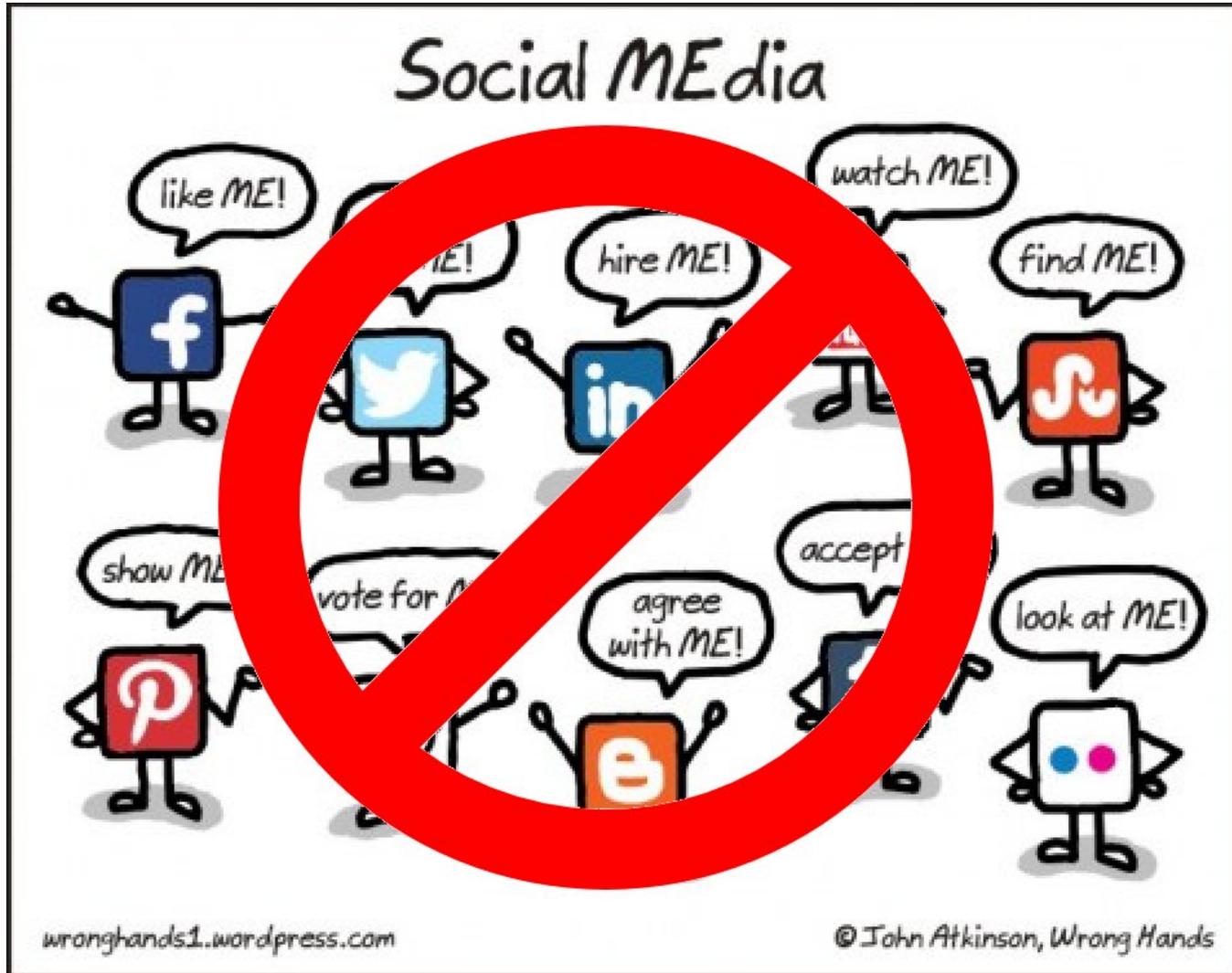


Physics 222, September 7

Key Concepts:

- Electrostatic potential energy
- The electric potential
- Conductors in electrostatics
- Capacitors

Please!



Remember Work!

Work is done by a force.

The work done by a force on an object is equal to the magnitude of the force multiplied by the distance the object moves in the direction of the force.

Notation: $\Delta W = \mathbf{F} \cdot \Delta \mathbf{r}$

In one dimension:

Work done on an object by a constant force:

$$W = F_x (x_f - x_i)$$

Work done on an object by a variable force:

$W = \sum_{x_i}^{x_f} F_x \Delta x$, as Δx becomes infinitesimally small.

Work is a scalar, a number with units.

Work can be positive or negative.

Electrostatic potential energy

An electric field E exerts a force $F_{el} = qE$ on a charge q .

An external force $F_{ext} = -qE$ can cancel the electric force.

The electric field is a conservative field.

The work done on a charge q by an external force $F_{ext} = -qE$ when moving it (with constant speed) in the presence of an electric field changes the **electrostatic potential energy** U of the charge.

Notation: $\Delta U = -qE \cdot \Delta r$.

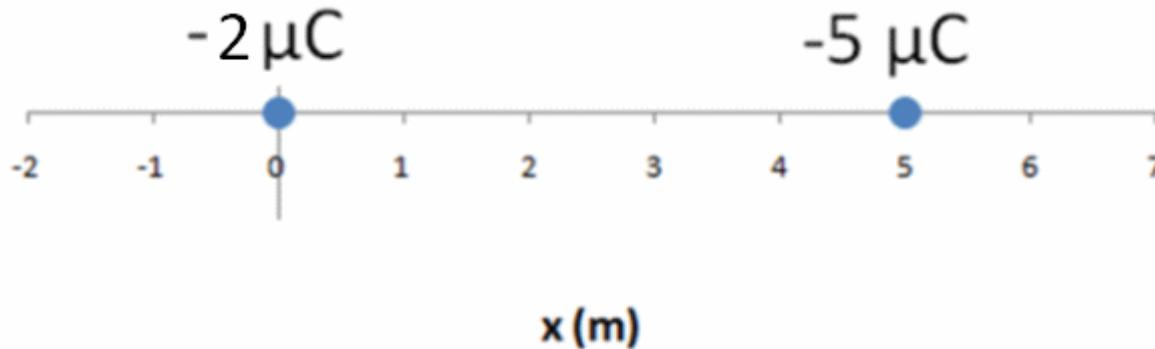
The electrostatic potential energy of a point charge q_2 when it is brought from infinity to a distance r away from another point charge q_1 is

$$U = k_e q_1 q_2 / r$$

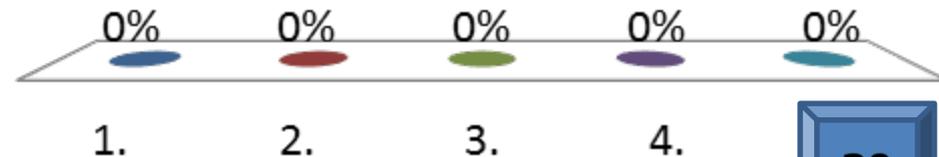
The **principle of superposition** holds. To find the **total electrostatic potential energy** of a collection of point charges, **sum over all pairs**.

For 3 charges: $U = k_e q_1 q_2 / r + k_e q_1 q_3 / r + k_e q_2 q_3 / r$

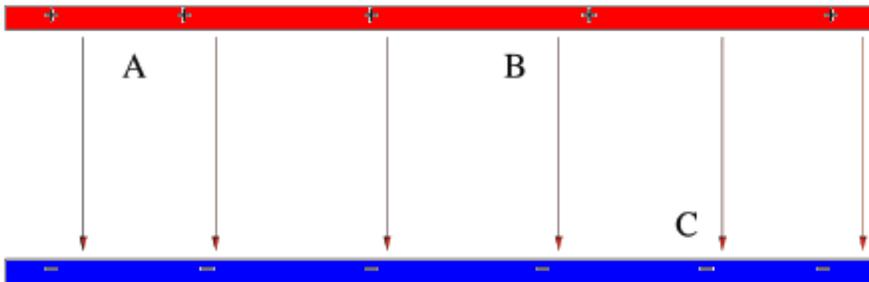
What is the electrostatic potential energy of a $-5\mu\text{C}$ charge located at $x = 5\text{ m}$ and a $-2\mu\text{C}$ charge located at the origin?



1. 0.092 J
2. 0.009 J
3. 0.018 J
4. -0.0018 J
5. 1.6×10^4 J



Consider a uniform electric field, for example the field inside a parallel plate capacitor, as shown. If an **electron** is taken from location A to location B to location C, how does its potential energy change?



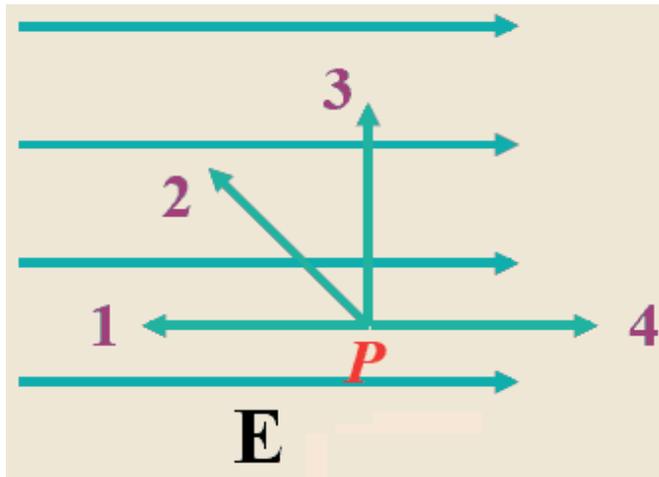
Hint: $\Delta U = -q\mathbf{E} \cdot \Delta \mathbf{r}$.

\mathbf{E} and $\Delta \mathbf{r}$ point in the same direction, so $\mathbf{E} \cdot \Delta \mathbf{r}$ is positive.

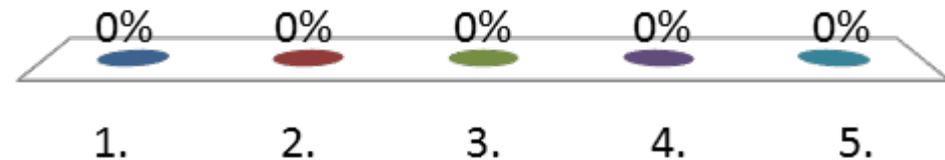
1. It decreases.
2. **It increases.**
3. It does not change.



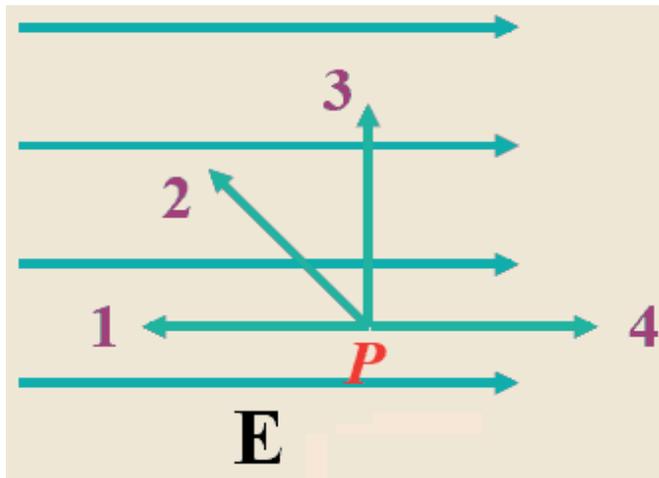
Which requires zero work, to move a **positive** point charge from point **P** to point **1**, **2**, **3**, or **4**? All those points are the same distance from P.



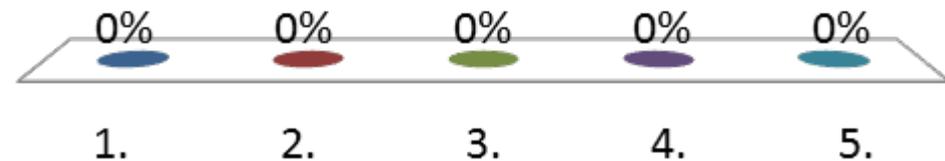
1. $P \rightarrow 1$
2. $P \rightarrow 2$
3. **$P \rightarrow 3$**
4. $P \rightarrow 4$
5. All require the same amount of work.



Which requires the **most positive work** done by an **external force**, to move a **positive** point charge from point **P** to point **1, 2, 3, or 4**? All those points are the same distance from P.



1. **P \rightarrow 1**
2. P \rightarrow 2
3. P \rightarrow 3
4. P \rightarrow 4
5. All require the same amount of work.



Electrostatic potential

Potential difference or voltage ΔV :

$$\Delta V = \Delta U/q$$

The voltage between two points is the **change in the electrostatic potential energy** of a test point charge q when it moves between those points, divided by q .

Units: **Volt (V) = Joule/Coulomb (J/C)**.

The potential of a point charge q' a distance r away from the charge:

$$V(\mathbf{r}) = q'/(4\pi\epsilon_0 r) = k_e q'/r.$$

The potential $V(\mathbf{r})$ is a scalar. It only depends on the position \mathbf{r} .

Potential due to many source: $V(\mathbf{r}) = V_1(\mathbf{r}) + V_2(\mathbf{r}) + V_3(\mathbf{r}) + \dots$

We add numbers (with units), not vectors.

Location P is equidistant from the two charges of an electric dipole. The voltage at P is

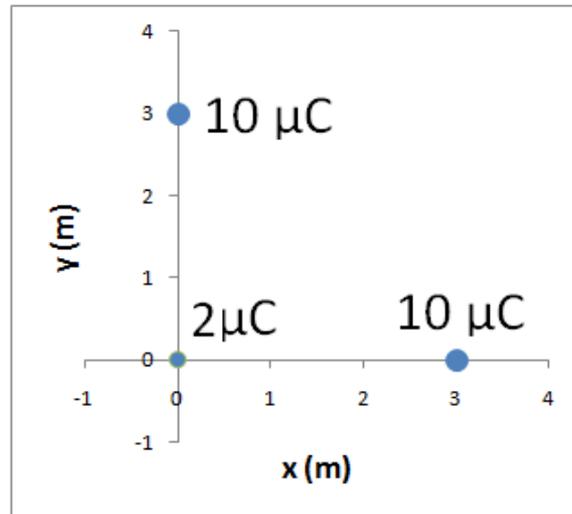
P



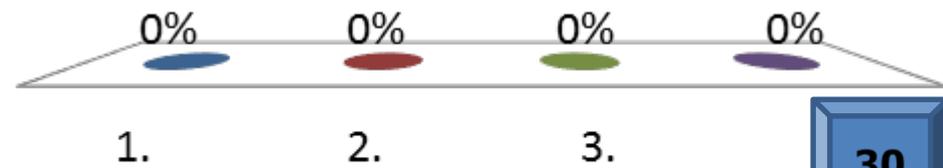
1. positive.
2. negative.
3. zero.



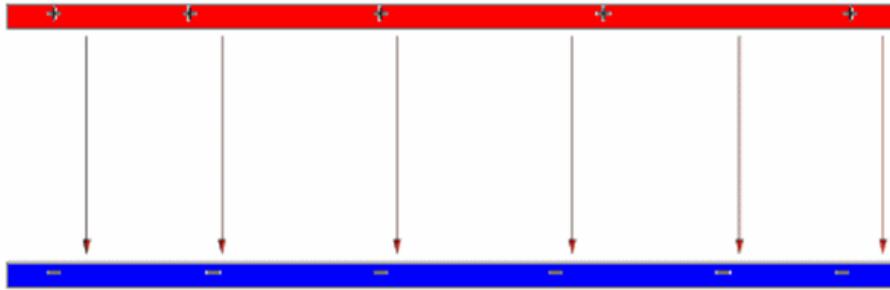
Two $10\ \mu\text{C}$ charges are located at $(x, y) = (0, 3\text{m})$ and $(3\text{m}, 0)$, respectively. What is the potential due to those two charges at the location of the $2\ \mu\text{C}$ charge at the origin?



1. $3 \cdot 10^4\ \text{V}$
2. $6 \cdot 10^4\ \text{V}$
3. $4.2 \cdot 10^4\ \text{V}$
4. $6 \cdot 10^{-2}\ \text{V}$



Assume that the potential difference between the positive and negative plate is 100 V. A proton travels from the positive to the negative plate.



Hint:
change in kinetic energy
+ change in potential energy
= 0

1. Its kinetic energy increases by $1.6 \times 10^{-17} \text{ J} = 100 \text{ eV}$.
2. Its kinetic energy decreases by $1.6 \times 10^{-17} \text{ J} = 100 \text{ eV}$.
3. Its kinetic energy does not change, only its potential energy changes.

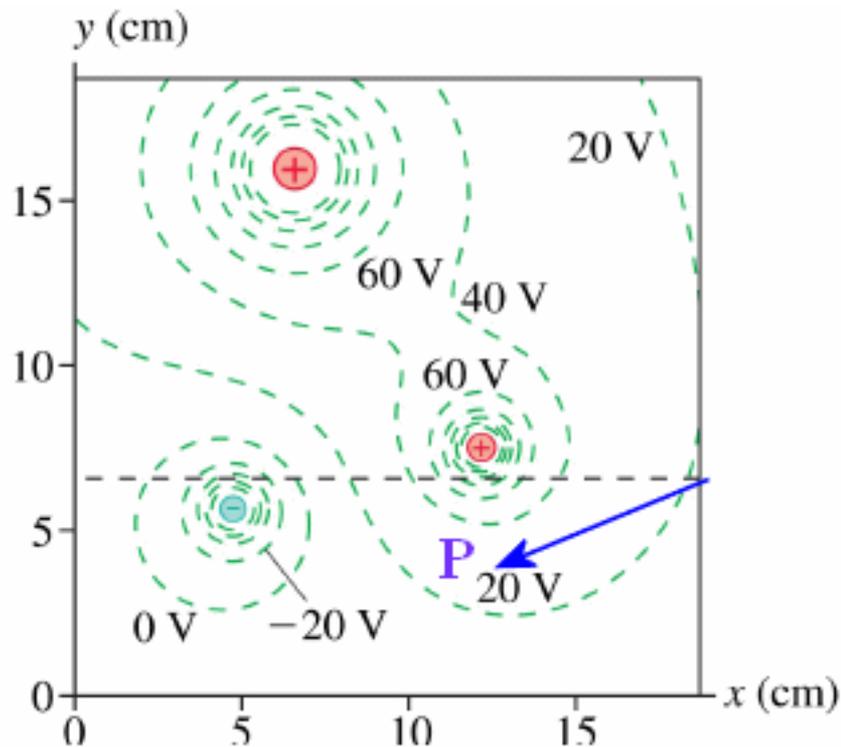


Visualizing the potential

Equipotential surfaces (contour plots).

- We can describe the electric potential pictorially with **equipotential surfaces** (contour plots).
- Each surface corresponds to a different fixed value of the potential.
- Equipotential lines are lines connecting points of the same potential.
- Equipotential lines are always perpendicular to field lines, and therefore perpendicular to the force experienced by a charge in the field.
- **If a charge moves along an equipotential line, no work is done**; if a charge moves between equipotential lines, work is done.

The graph below shows a contour map of the equipotential surfaces due to 3 point charges. Estimate the magnitude and direction of the electric field at point P.



1. ~ 20 V/m up
2. ~ 800 V/m down
3. ~ 2000 V/m to the right
4. ~ 40 V/m down
5. ~ 1000 V/m to the left

0% 0% 0% 0% 0%

1. 2. 3. 4.

30

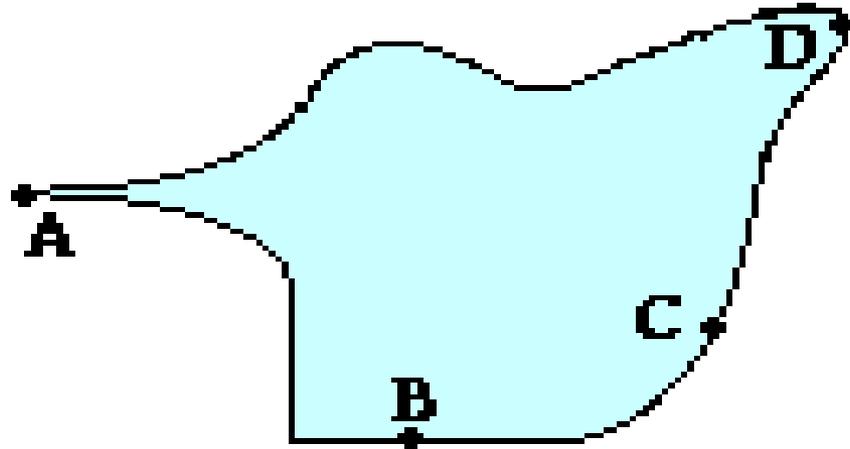
Conductors in electrostatics

In **electrostatic equilibrium** a conductor has the following properties.

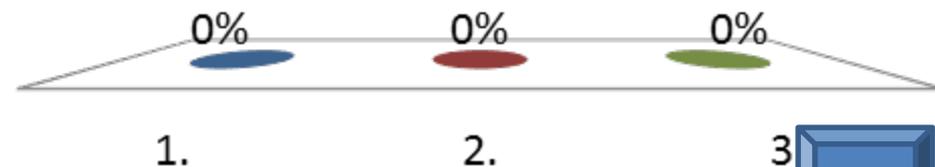
- Any excess charge resides on the surface of the conductor.
- The electric field is zero within the solid part of the conductor. The whole conductor is at the same potential.
- The electric field at the surface of the conductor is perpendicular to the surface.
- Charge accumulates, and the field is strongest on pointy parts of the conductor.

A conductor shields its interior from any outside electric fields.

A diagram of an irregularly shaped charged conductor is shown below. Four locations along the surface are labeled A, B, C, and D. Rank these locations in **increasing order of the strength** of the electric field just outside the surface, beginning with the smallest electric field



1. **$B < C < D < A$**
2. The field is the same everywhere just outside the surface.
3. $B = C < D = A$



Capacitors

A capacitor is a device for storing separated charge.

Any two conductors separated by an insulating medium form a capacitor.

Capacitance: $C = Q/V$

Q = amount of charge separated, V = voltage across the conductors.

C depends on the geometry of the device.

Parallel plate capacitor: $C = \epsilon A/d$

A = area of plates, d = plate separation, $\epsilon = \kappa\epsilon_0$,

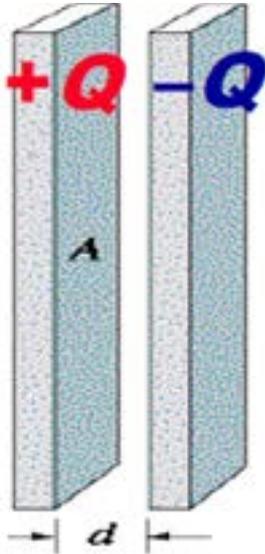
κ = dielectric constant of the material between the plates

Capacitors also store energy.

Electrostatic potential energy stored in the capacitor:

$$U = (1/2)QV = (1/2)(Q^2/C) = (1/2)CV^2$$

A parallel-plate capacitor initially is connected to a battery and the plates hold charge $\pm Q$. **The battery is then disconnected.** If the plate spacing is now doubled, what happens?



1. the voltage decreases
2. the voltage increases
3. the charge decreases
4. the charge increases
5. both voltage and charge change