Module 12

Key Concepts:

• Nuclear properties
• Nuclear decay
• Nuclear models
• Nuclear energy
Nuclear properties

A nucleus is a quantum particle made from protons and neutrons. We cannot track these particles inside the nucleus.

Notation: $^{A}_{Z}X$

$A = \# \text{ of nucleons}, \ Z = \# \text{ of protons}, \ X = \text{chemical symbol}$

Radius: $R = R_{0}A^{1/3}$ \quad $R_{0} = 1.2 \times 10^{-15} \text{ m}$

Classical picture
Interactions

The **long-range electrostatic force** cause protons to repel each other. The **short range nuclear or strong force** holds the nucleus together. It is **charge independent** and acts equally on neutrons and protons.

Range of nuclear force: $D_0 \sim 4$ proton diameters

The **nuclear binding energy** is the amount of energy needed to completely separate a nucleus into its component neutrons and protons.

Short range nuclear force $\rightarrow$ no super large nuclei
Protons are repelled by all the other protons.
Nucleons are attracted only by neighbors within $D_0$.

In super large nuclei:
**Electrostatic repulsion** $> \text{ nuclear attraction}$

Does this graph make sense?
Which of the following is not true?

1. The nuclear force has a short range, of the order of nuclear dimensions.
2. A nucleon in a large nucleus interacts via the nuclear force only with nearby nucleons, not with ones far away in the nucleus.
3. The nuclear force does not depend on the electric charge.
4. All of these statements are true.
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2. A nucleon in a large nucleus interacts via the nuclear force only with nearby nucleons, not with ones far away in the nucleus.

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4. All of these statements are true.

✓ 4. All of these statements are true.
Nuclear models

Shell model:
Confinement leads to energy quantization.

Energy levels can be grouped into shells. There are shells for protons and for neutrons. Nucleons fill the lowest available energy levels allowed by the Pauli exclusion principle.

In stable nuclei the highest energy protons have roughly the same total energy as the highest energy neutrons.

This leads to the “valley of stability”.
The Pauli exclusion principle is responsible for a shell structure in nuclei, similar to the shell structure in atomic physics, where the noble gases have especially large ionization energies.

Nuclei with magic neutron number \( N = 2, 8, 20, 28, 50, 82, 126 \) or magic proton number \( Z = 2, 8, 20, 28, 50, 82 \) have a larger binding energy per nucleon than neighboring nuclei and are called magic. When \( N \) and \( Z \) are both magic the binding energy per nucleon is especially large, and the nuclei are called doubly magic.
E = mc²

Binding energy formula

Using **nuclear masses**:

\[ B(Z,N) = c^2(Zm_p + Nm_n - M_{\text{nucl}}(Z,N)) \]

(nuclear masses are usually given in units of MeV/c²)

Or, using **atomic masses**:

\[ B(Z,N) = c^2(Zm_H +Nm_n - M_{\text{atom}}(Z,N)) \]

(atomic masses are usually given in atomic mass units u)

1 u = 931.494 MeV/c²
Nuclear decay

Decay is a quantum process. All we can know is the decay probability.

decay constant: \( \lambda = \text{decay probability per unit time} \)
mean lifetime: \( \tau = 1/\lambda \)
half-life: \( t_{1/2} = \tau \ln 2 = \ln 2/\lambda \)

The half-life is the time it takes for half the nuclei to decay.

# of nuclei left after time t: \( N(t) = N_0 \exp(-\lambda t) \)
decay rate at time t: \( R(t) = R_0 \exp(-\lambda t) \)

Decay modes:

alpha decay:

\[
\begin{array}{c}
222_{86}\text{Ra} \rightarrow 218_{84}\text{Po} + 4_2\text{He}
\end{array}
\]

beta decay:

\[
\begin{array}{c}
\text{Carbon-14} \rightarrow \text{Nitrogen-14} + \text{Antineutrino} + \text{Electron}
\end{array}
\]

\[
\begin{array}{c}
\text{Carbon-10} \rightarrow \text{Boron-10} + \text{Neutrino} + \text{Positron}
\end{array}
\]

gamma decay:
Nuclear energy

E = mc²: Whenever a system loses energy, it loses mass.

Energy Source: Chemical Fission Fusion

Efficiency (E/mc²): 3 * 10⁻⁸% 0.002% 0.4%